GRADE DE CONTRADICȚIE PENTRU ONTOLOGII DE DOMENIU REPREZENTATE PRIN LOGICI FUZZY

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Rezumat: Scopul principal al acestui scurt articol este acela de a analiza câteva posibile grade de contradicție între două reguli exprimate în logici fuzzy. Pe această bază, se propun implementări software utile, în special în cercetări referitoare la optimizarea multi-obiectiv automată a unor sisteme complexe de calcul. În particular, calculul automat al gradelor de contradicție aferente unor ontologii de domeniu reprezentate prin reguli în logici fuzzy – precum cele propuse în [4] - este de un interes științific cert.

Cuvinte cheie: Reguli reprezentate în logici fuzzy, metrici de similaritate/disimilaritate, grade de contradicție, arhitectura calculatoarelor.

Abstract: The main aim of this short note is to analyze some degrees of contradiction for two CNF fuzzy rules and to suggest some possible useful software implementations, especially for Multi-Objective Optimization through Automatic Design Space Exploration applied in Advanced Computer Architecture research. Particularly, automatically calculating the degrees of contradiction for domain ontologies represented using fuzzy logic rules - like that proposed in [4] - would be of certain scientific interest.

Keywords: Fuzzy logic rules, Similarity/Dissimilarity Metrics, Degree of Contradiction, Computer Architecture

1. Introduction

We are considering two classical logical rules represented as below:

R_1: IF $A_1=5$ AND $A_2=9$ THEN $O=1$, (1)

R_2: IF $A_1=5$ AND $A_2=9$ THEN $O=0$ (2)

It's obvious that these rules are contradictory because the antecedents are identical and the outputs are different. This contradiction metaphor will be further used for proposing degree of contradiction metrics for fuzzy logic rules.

Now we are considering two fuzzy rules R_i and R_j represented in the Conjunctive Normal Form (CNF). Both rules have the same output O (consequent). The rules are defined as the followings:

R_i: IF $A_1=V(A_1^i)$ AND $A_2=V(A_2^i)$ AND … AND $A_m=V(A_m^i)$ THEN $O=V(O_i)$, (3)

where $V(A_1^i)$, $V(A_2^i)$, …, $V(A_m^i)$ are gradual discrete functions (or fuzzy sets memberships functions) associated to the linguistic variables A_1, A_2, …, A_m respectively, and $V(O_i)$ represents a fuzzy sets memberships function of the linguistic variable O.

R_j: IF $A_1=V(A_1^j)$ AND $A_2=V(A_2^j)$ AND … AND $A_m=V(A_m^j)$ THEN $O=V(O_j)$, (4)

where $V(A_1^j)$, $V(A_2^j)$, …, $V(A_m^j)$ are fuzzy sets memberships functions of the linguistic variables A_1, A_2, …, A_m respectively, and $V(O_j)$ represents a fuzzy sets memberships function of the linguistic variable O.

2. Degrees of contradiction between two fuzzy rules

In this context, it makes sense to define a degree of contradiction between the fuzzy rules R_i and R_j. In [2] it is proposed a definition for the degree of contradiction between two fuzzy rules having the same antecedents and different consequent (output) values, as the following one:

$$C(R_i, R_j)= S(I_i, I_j) \cdot D(V(O_i), V(O_j)) = \frac{\sum_{k=1}^{m} S(V(A_k^i), V(A_k^j))}{m} \cdot D(V(O_i), V(O_j)) \quad (5)$$

where $S(V(A_k^i), V(A_k^j))$ represents a similarity metric between two fuzzy sets belonging to the fuzzy rule antecedents and D represents a dissemblance (dissimilarity) metric between two fuzzy sets representing the fuzzy rules' consequent. Similarity represents a metric of the non-contradiction of two fuzzy sets membership functions. When S and D are normalized (belonging to [0, 1] interval) it might be considered that D(x, y)=1-S(x, y), $\forall$ x,y$\in$X (set of fuzzy memberships).

$$\text{We noted } S(I_i, I_j)= \frac{\sum_{k=1}^{m} S(V(A_k^i), V(A_k^j))}{m} \quad (6)$$

where I_i and I_j are the fuzzy rules antecedents represented as the vectors $I_i = [A_1=V(A_1^i), A_2=V(A_2^i), \ldots, A_m=V(A_m^i)]$ and $I_j = [A_1=V(A_1^j), A_2=V(A_2^j), \ldots, A_m=V(A_m^j)]$. Of course, other aggregation formulas would be possible (harmonic mean, etc.).

Similarity metrics must fulfill symmetry (S(x, y)=1 $\Leftrightarrow$ x=y), reflexivity (S(x,y) = S(y,x)), and sensitivity mathematical properties. Also it is necessary that $\forall$ x,y,z$\in$X $\rightarrow$ S(x,z) $\leq$ S(x,y) + S(y,z).

It is obvious that $C(R_i, R_j)=1$ if and only if $S(I_i, I_j)=1$ and $D(V(O_i), V(O_j))=1$.

In [3] the authors proposed some degrees of contradiction for two fuzzy sets identified as their membership functions and determined some of their mathematical properties. In [2] the authors proposed a dissimilarity metric called dissemblance index considering two fuzzy sets defined on ***R*** set (thus the membership function is *m:**R**→[0, 1]*). They are defining the dissimilarity between two fuzzy sets – noted by us with $D(V(A_k^i), V(A_k^j)) \in [0, 1]$ - as a normalized positive difference between the areas defined by the corresponding membership functions (see the corresponding formulas in [2] pages 7-8). Taking into account that *D* is normalized, it is obvious that S can be defined as: $S(V(A_k^i), V(A_k^j)) = 1 - D(V(A_k^i), V(A_k^j))$.

Instead of defining dissimilarity between two fuzzy sets we propose - as an simpler computing alternative - to calculate, in a run-time manner, the dissimilarity between the concrete crisp values $Val(A_k^i)$, $Val(A_k^j)$ associated to the linguistic variables A_k^i, A_k^j respectively. Considering the associated membership functions MA_k^i: ***R**→[0, 1]* and MA_k^j: ***R**→[0, 1]* and noting them $MA_k^i(Val(A_k^i))= VA_k^i$, $MA_k^j(Val(A_k^j))= VA_k^j$, the dissimilarity between the crisp values $Val(A_k^i)$ and $Val(A_k^j)$ can be defined, for example, as $D(Val(A_k^i), Val(A_k^j))= \left|VA_k^i - VA_k^j\right|$. In this case the dissimilarity between two fuzzy rules antecedents can be calculated, similar with formula (6), as $D(I_i, I_j)= \frac{\sum_{k=1}^{m} \left|VA_k^j - VA_k^i\right|}{m}$. If the dissimilarity value is over a certain threshold, the two rules might be too contradictory.

In [1] the authors are proposing other similarity metrics based on the distance between two fuzzy sets (A, B). According to them some of the possible relationships between distance d(A,B) and similarity S(A,B) are the followings:

$S(A,B)=1/(1+d(A,B))$ or $S(A,B)=e^{-ad(A,B)}$, where *a* represents the steepness measure.

Distances like (normalized) *Hamming* distance, (non-normalized) *Euclidian* distance - $d_E(A,B)=d_E$ – and others are proposed (see the concrete formulas in [1] page 195). For example: if A and B are two fuzzy sets with the corresponding membership functions m_A and m_B, the normalized *Hamming* distance is defined as $d_E(A,B)= \frac{1}{n}\sum_{i=1}^{n}\left|m_A(x_i) - m_B(x_i)\right|$ (7).

For normalizing *Euclidian* distance d_E between two fuzzy sets we might use as an algebraic "squashing" function like $d_E \rightarrow \frac{d_E}{\sqrt{1+d^2_E}}$, taking values in [0, 1] interval for any positive argument (d_E). Other "squashing" functions are possible (for example the sigmoid activation function given by the well-known expression *1/(1+exp(-x))*).

We propose - as another possible distance metric - the distance between the centers of mass of the planar region defined by the membership functions of fuzzy sets A and B. Therefore:

$$d(A,B) = \sqrt{(x_{BG} - x_{AG})^2 + (y_{BG} - y_{AG})^2}, \quad (8)$$

where:

$$x_G = \frac{\sum_{i=1}^{n} x_i m_A(x_i)}{\sum_{i=1}^{n} m_A(x_i)} \text{ and } y_G = \frac{\sum_{i=1}^{n} m^2{}_A(x_i)}{2\sum_{i=1}^{n} m_A(x_i)} \quad (9)$$

Pearson's distance (*d*), based on the correlation coefficient of two vectors (*r*), frequently used in image processing, might be another candidate (*d=1-r*). An open problem, at least for us, is the following: could be defined a contradiction degree between two CNF fuzzy rules having antecedent vectors that have common elements but are not identical?

In [4] it is proposed a set of fuzzy rules – implementing a speculative superscalar microprocessor design ontology - presented below:

- IF Number_of_Physical_Register_Sets IS *small/big* THEN Decode/Issue/Commit_Width IS *small/big;* 2 rules!
- IF SLVP_Size IS *small/big* THEN L1_Data_Cache IS *big/small;* 2 rules!
- IF SLVP_N IS *small* AND SLVP_Assoc IS *small* THEN SLVP_Size IS *big*

Due to some *apriori* feelings related to possible high degrees of contradiction, the authors avoid other possible (feasible) rules like:

- IF SLVP_N IS *big* AND SLVP_Assoc IS *big* THEN SLVP_Size IS *big;* it seems to make sense.

or

- IF SLVP_N IS *small* AND SLVP_Assoc IS *big* THEN SLVP_Size IS *medium*

These rules - acting as a Computer Architecture domain ontology - are integrated in their *Framework for Automatic Design Space Exploration* (FADSE, see http://code.google.com/p/fadse/) in order to develop hardware-software multi-objective optimizations for some advanced computing systems. More precisely, for example the bit flip mutation operator was extended to stochastically take into consideration the information provided by the outputs of the fuzzy rules after a fuzzification – inference – defuzzification process. Thus, the fuzzy logic rules are integrated into the genetic operators of their state of the art implemented evolutionary multi-objective optimization algorithms, with significant benefits in the algorithms' convergence speed and in the obtained solutions' quality, too [4, 5].

In fuzzy logic material implication (IF/THEN) has many definitions (*Zadeh, Lukasiewicz, Mamdani, Reichenbach* formulas are some of the most frequently used). The „best" fuzzy logic implication form is problem-dependent. An interesting open question might be: what is the best (most effective) fuzzy implication form for a certain application?

3. Conclusion

Computing the contradiction degrees for the presented set of fuzzy logic rules, starting from the presented approach – and using the corresponding concrete defined membership functions - would be helpful in order to avoid solutions quality degradation due to these (possible too contradictory) rules. Also, we need in our further developed mono-core and multi-core domain ontologies (example: for *Sniper* multicore simulator) to be sure that the contradiction degrees in a complex set of fuzzy rules are quite “acceptable” in order to maintain the optimization effectiveness. Finding some thresholds for acceptable degrees of contradiction is problem dependent. Finally, an optimal set of such fuzzy rules, for a certain optimization problem, is envisaged. More general, developing a software tool capable to automatically calculate the contradiction degrees of a set of (CNF) fuzzy rules - using different approaches, including those presented above - would be of interest, too. Particularly we are interested to develop and integrate such a tool in our *Framework for Automatic Design Space Exploration* in order to improve the optimization process through an adequate set of fuzzy logic rules that are implementing a Computer Architecture domain ontology.

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