Global Stability Analysis of Mechanical Prosthetic Finger Adaptive Control

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Abstract: This technical note presents a discussion on the dynamic modeling and analysis of the adaptive controller of a Mechanical Prosthetic finger with constraints mass (m) of the mechanical finger and constant friction (K_1) with variation in spring constraint (K_2) parameter during grasping action. For the design of the adaptive controller and assessment of adaptation gain, an underdamped second-order control system has been considered and variation of adaptation gain (γ) within certain pre-defined limits of system parameters, variation in the adaptation mechanism has been analyzed using Gradient method MIT rule. To ensure global stability along with the convergence on nonconformity of plant parameters Lyapunov Rule has been utilized towards closed-loop asymptotic tracking.

Keywords: Mechanical Prosthetic finger, Adaptive model, Gradient method MIT rule, Lyapunov Rule, Global Stability.

1. Introduction

In the field of prosthetic control, dynamic and mathematical modelling of a system, as well as its computation using a biological governor, has sparked a lot of interest (Rossini et al., 2010, Neogi et al., 2012, Neogi et al., 2016). Software-aided control system design (SACSD) includes a wide range of tools and numerical modelling for system design, dynamic system study and simulation, and performing practical trials on a given plant model to explore and anticipate the dynamic activities of a physical plant. Modeling and simulation are frequently related components of a dedicated environment, where detailed classes of models may be constructed and appropriate imitation methods can be used (Engeberg, 2013). This paper aims to govern the grasping action of fingers by looking at the controller adapted with an anticipated reference, a self-adaptable controller with adaptive mechanism using MRAC-MIT rule for resolving the action of finger prosthesis relative to mass-damper-spring system (Neogi et al., 2012, Neogi et al., 2016). As a postponement of the previous effort in the domain, this paper aims at governing the grasping.

Combining Controller parameters to perfect standards, which force the response to track the reference model, i.e. the swift response between fingers of the prosthetic arm with minor or no pulsations, the step response would give the anticipated enactment with less or no peak overshoot, optimum rise time, optimum settling time, optimum damping ratio, and zero steady state error. The transfer function is used to demonstrate reference behaviour. Because adaptive controllers based on the MIT rule do not guarantee convergence or stability (Neogi et al., 2016), this work also includes a Lyapunov stability analysis for closed loop asymptotic tracking.

MATLAB has recently achieved widespread pedagogical acceptability by incorporating a number of novel practical developments in the control engineering area, as well as a plethora of control-related toolboxes. The CACSD tools of MATLAB and Simulink are frequently extended and enriched through innovative model classes, new computational algorithms, innovative robust control system synthesis, dedicated graphical user interfaces (e.g., tuning of PID controllers or control related visualizations), and so on (MATLAB & Statistics Toolbox Release, 2015).

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2. Mechanical Finger Control Modelling using gradient based MRAC-MIT rule

Mechanical finger considered as a damped 2nd order differential equation,

$$f(t) = m\frac{d^2x}{dt^2} + K_1\frac{dx}{dt} + K_2 x(t) \quad (1)$$

Where *f(t)* = force applied to produce shift amongst fingers, m= mechanical finger mass, *x(t)*= shift between fingers, K_1= friction constant, K_2= spring constant. (Neogi et al., 2012).

For the analysis we are considering 2nd, 3rd and 4th set of transfer functions and a comparative analysis utilizing 1st set of transfer is already done (Neogi, B. et al. 2016) defined as;

Set 1: $G(s) = \frac{1}{0.21s^2 + 0.4038s + 0.0411}$

Set 2: $G(s) = \frac{1}{0.21s^2 + 0.1183s + 0.024}$

Set 3: $G(s) = \frac{1}{0.21s^2 + 0.0434s + 0.0299}$

Set 4: $G(s) = \frac{1}{0.21s^2 + 0.2369s + 0.0054}$ (2)

This work is generally concerned with the objectives outlined as;

Controlling the shift between fingers of prosthetic fingers using gradient method MIT rule, its adaptive tuning. The output stability analysis for the class of both nonlinear system using Lyapunov analysis. A linear approximation of nonlinear system about an equilibrium point is discussed and how the local stability holds for the system. A plant of higher order differential equation with PD controller in a closed loop system shows stability, here verified by Lyapunov method. Lately, Lyapunov method is used to design stable Model Reference Adaptive Controller.

Subsequently, input as applied force *f(t)= F(s)* to produce shift and output as movement among fingers $x(t) = X_p(s)$ in s-domain;

$$G_p(s) = \frac{X_p(s)}{F(s)} = \frac{\frac{1}{m}}{[s^2 + \frac{K_1}{m}s + \frac{K_2}{m}]} \quad (3)$$

Thus, system gain *c*, *a* & *b* is time constant and time variable and is represented as,

$$a = \frac{K_1}{m}; b = \frac{K_2}{m}$$

2nd order under damped system taken as reference model can be written as,

$$X_m(s) = G_m(s)R(s) = \frac{c_m}{[s^2 + a_m s + b_m]} \quad (4)$$

Tracking error, $e = X_p - X_m$ (5)

Adaptive law to trail the desired reference model in following way,

$$f = [S_1 r - S_2 x_p - \dot{x}_p] \quad (6)$$

Here, S_1,S_2,S_3 are the Adaptive feed forward-feedback controller parameter expressed in vector form as $\theta = (S_1, S_2, S_3)$ and $\delta = [r, -x_p, -\frac{d}{dt}x_p]^T$. Substituting eqn. (6) in time domain form of eqn. (3), then eqn. (7) will track the reference of eqn. (5),

$$\ddot{x}_p + (a + cS_3)\dot{x}_p + (b + cS_2)x_p = cS_1 r \tag{7}$$

Controller parameters taken as;

$$S_1 = \frac{c_m}{c}; S_2 = \frac{b_m - b}{c}; S_3 = \frac{a_m - a}{c} \tag{8}$$

From eqn. 8, it can be seen that *a*, *b* & *c* are unidentified and changeable, thus adaptive variation of each controller parameter S_1, S_2, S_3 can be estimated based on measurable variables. In a generalised form, MIT Rule is represented as (Engeberg, 2013),

$$\frac{d\theta}{dt} = -\gamma e \frac{de}{d\theta}$$

Here, γ denotes adaptation gain.

Henceforth, Cost function, $(\theta) = \frac{1}{2}e^2(\theta)$, which has to be curtailed.

Using $s(.) = \frac{d}{dt}(.)$ from differential eqn. (7) the result is,

$$X_p = \frac{cS_1}{s^2 + (a + cS_3)s + (b + cS_2)} r \tag{9}$$

The sensitivity derivatives $\frac{\delta e}{\delta \theta}$; given as,

$$\frac{\delta e}{\delta S_1} = \frac{\delta}{\delta S_1}(X_p - X_m) = \frac{c}{s^2 + (a + cS_3)s + (b + cS_2)} r \, ; \, \frac{\delta e}{\delta S_2} = -\frac{c}{s^2 + (a + cS_3)s + (b + cS_2)} X_p$$

$$\frac{\delta e}{\delta S_3} = -\frac{sc}{s^2 + (a + cS_3)s + (b + cS_2)} X_p \tag{10}$$

Tracking path can be modelled using eqn. (8) as $S_1 c = c_m, (a + cS_3) = a_m, (b + cS_2) = b_m$

From eqn. (8) the update law for $\theta = (S_1, S_2, S_3)$ are given by,

$$\frac{dS_1}{dt} = -\gamma e \frac{c_m}{[s^2 + a_m s + b_m]}$$

$$\frac{dS_2}{dt} = \gamma e \frac{c_m}{[s^2 + a_m s + b_m]} X_p$$

$$\frac{dS_3}{dt} = \gamma e \frac{c_m s}{[s^2 + a_m s + b_m]} X_p$$

Hence, γ, i.e. adaptive gain is written as

$$\gamma = \gamma \frac{c}{c_m} \tag{11}$$

3. Outcomes and analysis

The constraints linked to the sets of transfer function are given in Table I. as, prosthetic finger mass (m) = 210 gm [3]. The adaptation gain γ is sculpted, the subsequent effect on the system's reaction is inspected. The values of the test with gain 0.1, 2.4 and 5 are shown in Figure 2.

Table I

Transfer function	Parameters	
	K_1= friction constant	K_2 = spring constraint
Set 2	0.11935	0.0245
Set 3	0.0434	0.0299
Set 4	0.23695	0.0054

The Figure 1(i) displays a block illustration of prosthetic finger adaptive controller with tuning mechanism. A square signal with unit amplitude and a time period of 20 seconds is applied and the equivalent time response is shown in Figure 1(b).

Considering reference model transfer function;

$$G_m(s) = \frac{2.460592}{s^2 + 2.6667s + 2.460592} \tag{12}$$

The subsequent comments can be made from these results:

Increase in the gain rises the oscillation frequency and also result in the rise of overshoot.

The error variance is shown in Figure 2(ii) - Figure 2(iv).

Due to addition of PD controller across the plant adaptation law, the MRAC structure changes as the following form, in S-domain yields

$$F(s) = S_1[\{K_p + sK_d\}\{R(s) - X_p(s)] - S_2 X_p(s) - S_3 s X_p(s) \tag{13}$$

Substituting eqn. (13) into eqn. (4) yields

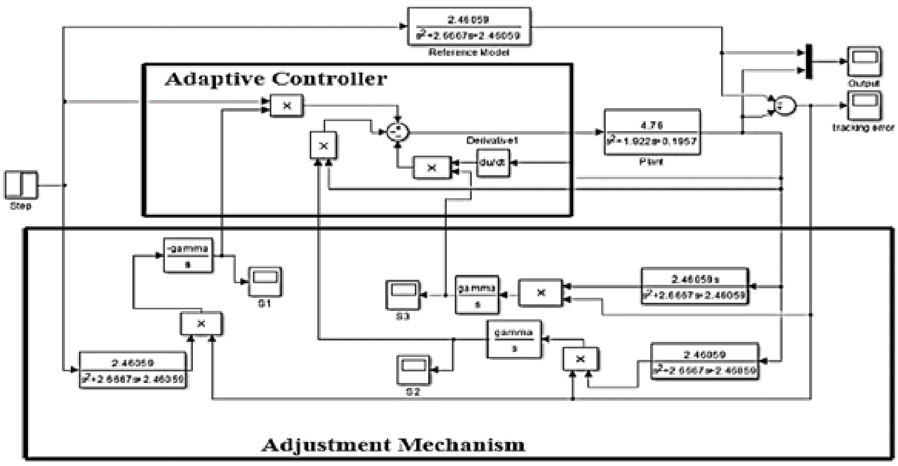


Figure 1(i). Simulink model of finger prosthesis

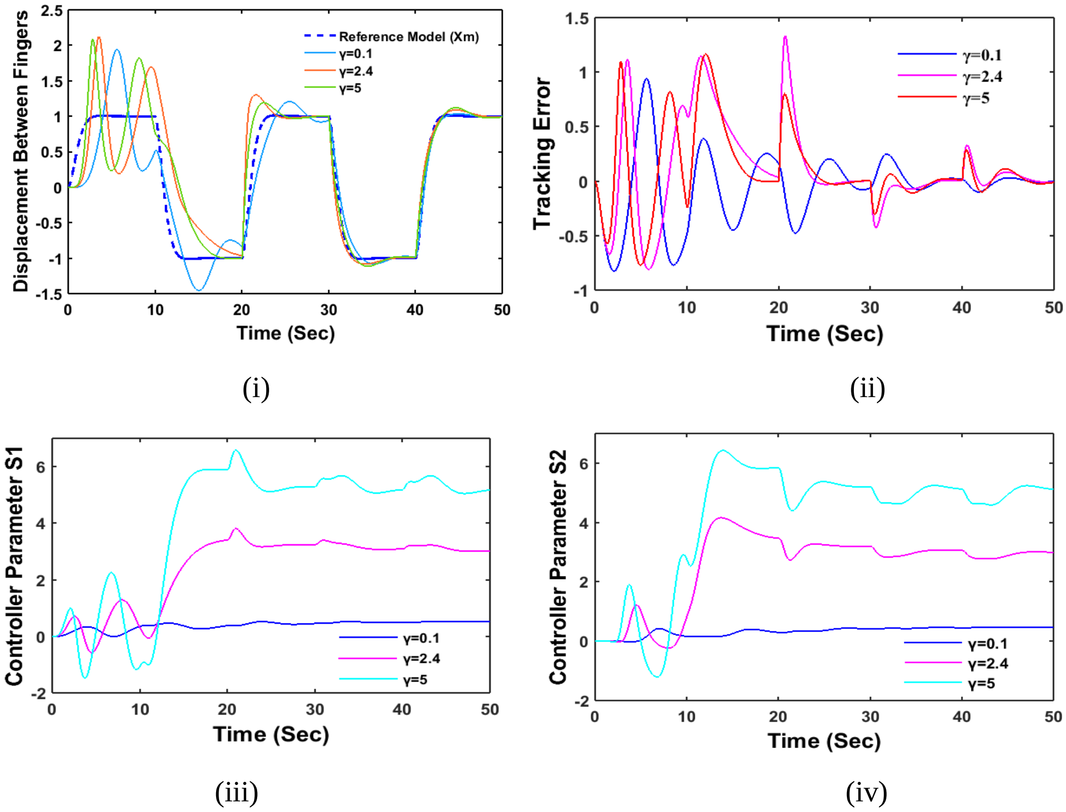


Figure 2(i). Time response corresponding to square input, (ii)-(iv) the error between the reference and the process

$$\{s^2+as+b\}X_p(s)=\{K_p+sK_d\}\{R(s)-X_p(s)\}-cS_2X_p(s)-cS_3sX_p(s) \tag{14}$$

Thus we can represent eqn.14 as;

$$\frac{X_p(s)}{R(s)}=\frac{c(K_p+sK_d)S_1}{s^2+(a+cS_3+cS_1K_d)s+(b+cS_2+cK_pS_1)} \tag{15}$$

Eqn.15, depicts a modified plant transfer function with a fixed gain PD controller placed over the adaptive controller block to eliminate oscillations caused by instability.

The major goal is to get the plant as close to the model as possible, the plant's parameters are set close enough to the ideal model values so the plant's characteristics match the model's., i.e.:

$$s^2+(a+cS_3+cS_1K_d)s+(b+cS_2+cS_1K_p)\approx[s^2+a_ms+b_m] \tag{16}$$

The sensitivity derivatives S_1, S_2, S_3 for each value of system controller parameters are calculated using the global MRAC-MIT rule. written as;

$$\frac{\delta e}{\delta S_1}=\frac{c(K_p+sK_d)\{R(s)-X_p(s)\}}{[s^2+a_ms+b_m]}\quad \frac{\delta e}{\delta S_2}=-\frac{c}{[s^2+a_ms+b_m]}X_p(s)$$

$$\frac{\delta e}{\delta S_3}=-\frac{cs}{[s^2+a_ms+b_m]}X_p(s) \tag{17}$$

Thus, the updated law for each of the controller parameters $\theta=(S_1,S_2,S_3)$ is as follows

$$\frac{dS_1}{dt}=-\gamma_1 e\frac{\delta e}{\delta S_1}=\gamma_1 eG_m(s)(K_p+sK_d)\{R(s)-X_p(s)\} \tag{18}$$

Similarly; $$\frac{dS_2}{dt} = -\gamma_2 e \frac{\delta e}{\delta S_2} = \gamma_2 e G_m(s) X_p(s) \quad (19)$$

And $$\frac{dS_3}{dt} = -\gamma_3 e \frac{\delta e}{\delta S_3} = \gamma_3 e G_m(s) s X_p(s) \quad (20)$$

Where adaptive gain for each controller parameters (S_1, S_2, S_3) is

$\gamma_1 = \gamma_1\left(\frac{c}{c_m}\right)$ And $\gamma_2 = \gamma_2\left(\frac{c}{c_m}\right); \gamma_2 = \gamma_2\left(\frac{c}{c_m}\right)$

Adaptive gain is first implemented in the prior segment, and the effect on reaction time is investigated. Steering the same trial as in the previous section with gains of 0.1, 2.4, and 5 yields the results shown in Figure 4 (i)-(v). Time reaction, tracing errors between the plant and the reference system, and overseeing constraints are some of the outcomes of the investigation Figure 5 (ii)-(v) illustrates the tracking error among the reference scheme and the process over time. Similarly, MRAC without a PD controller has larger overshoot values and takes longer to stabilise the response. However, there is no severe overshoot, and it takes less time for the reaction to be level. The reference is chosen so that it responds quickly to a step input.

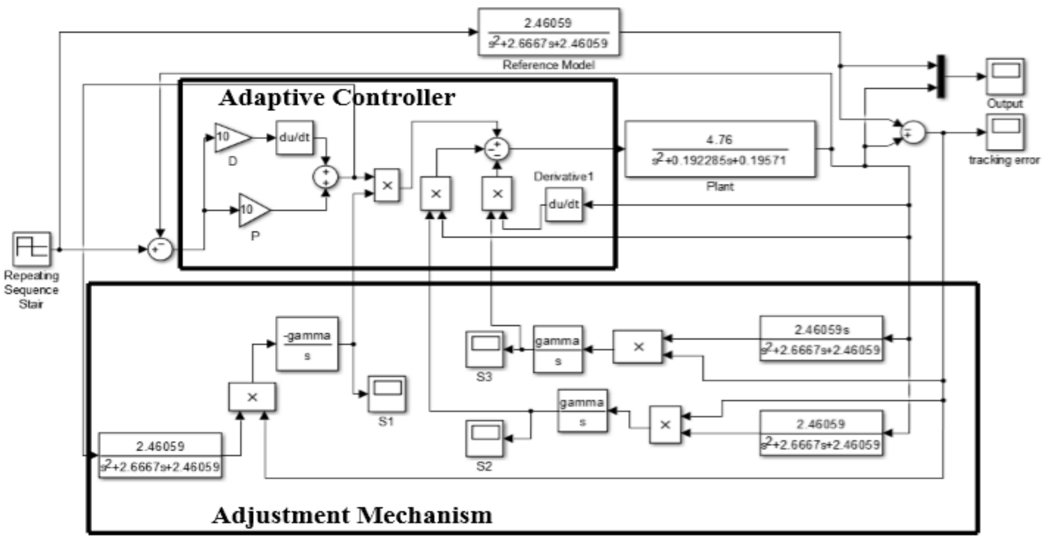
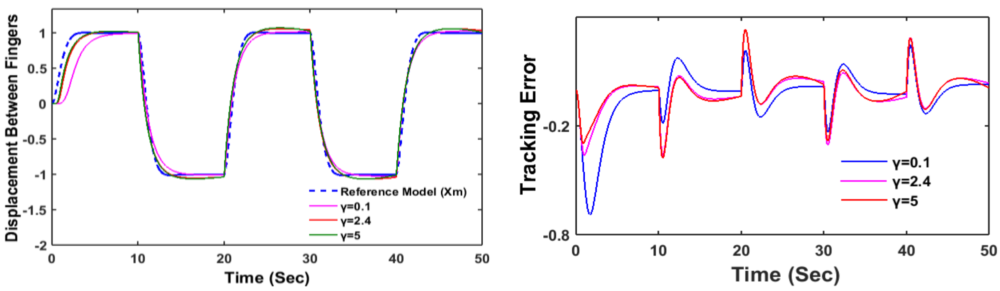
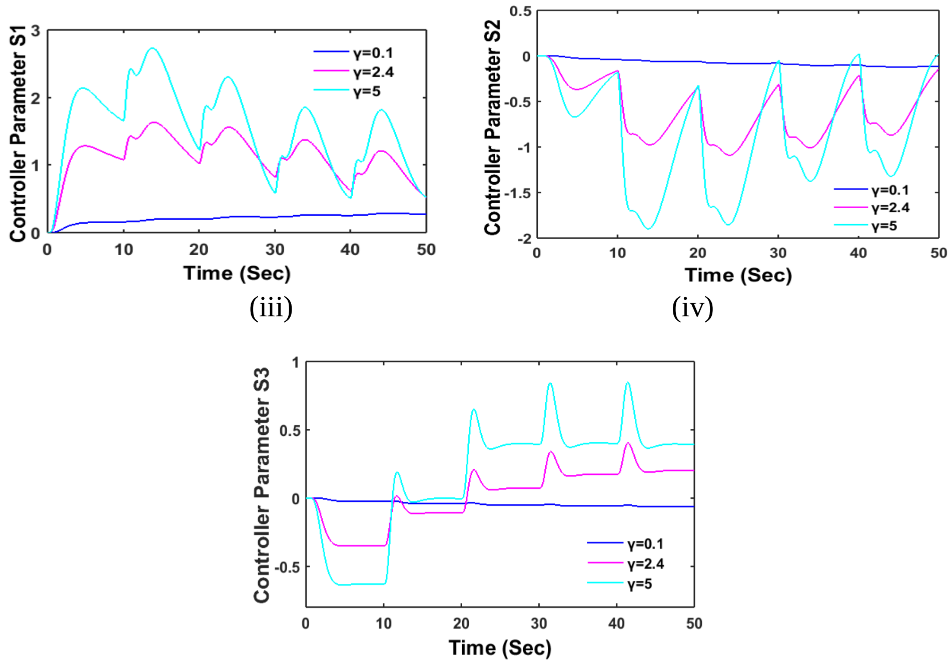


Figure 3. Simulink module of finger prosthesis with PD controller in the Adaptive module



(i) (ii)



(v)

Figure 4. Time response corresponding square input, (ii)-(iv). Error Tracking amongs the reference model and the process over time corresponding to fixed gain PD controller

$$G_m(s) = \frac{2.6130}{s^2 + 2.28558s + 2.6130} \tag{21}$$

The reference having a relaxing time of 3.5 sec, a damping factor 0.707 and a percentage overshoot of 0.63%.

4. Global Stability Analysis utilizing Lyapunov Criterion

Lyapunov-based design can be used to keep the adaptive system stable. The result is obtained by subtracting eq. (4) from eq. (5).

$$(\ddot{x}_p - \ddot{x}_m) + (a + cS_3)\dot{x}_p - a_m x_m + (b + cS_2)x_p - \dot{b}_m x_m = cS_1 r - c_m r \tag{22}$$

Let,

$$(a + cS_3) - a_m = a_1 \,;\, (b + cS_2) - b_m = b_1 \,;\, cS_1 - c_m = c_1$$

Then; Eq. (13) can be represented as,

$$\ddot{e} + a_m \dot{e} + b_m e = c_1 r - b_1 x_p - a_1 \dot{x}_p \tag{23}$$

Lyapunov function entrant chosen as,

$$V(e, \dot{e}, c_1, b_1, a_1) = \dot{e}^2 + b_m e^2 + \frac{1}{\gamma_1} c_1^{\,2} + \frac{1}{\gamma_2} b_1^{\,2} + \frac{1}{\gamma_3} a_1^{\,2}$$

Then its derivative is; $\dot{V} = 2\dot{e}\ddot{e} + 2b_m e\dot{e} + \frac{2}{\gamma_1} c_1 \dot{c}_1 + \frac{2}{\gamma_2} b_1 \dot{b}_1 + \frac{2}{\gamma_3} a_1 \dot{a}_1$ (24)

This gives,

$$\dot{S}_1 = -\alpha \dot{e} r \,;\, \dot{S}_2 = -\alpha \dot{e} x_p \,;\, \dot{S}_3 = \alpha \dot{e} \dot{x}_p$$

α is taken as, $\alpha = \dfrac{\gamma}{c}$

The negative sign of the derivative denotes the design of a steady adaptive controller, and eqn. (23) denotes the attainable adaptive law of controller. Adaptive gain γ is examined first in the previous segment, followed by the impact on time response. The effects of altering the gain levels in the same way are shown in Figure 5. The system characteristics that correlate to the approximate overshoot and settling time are compared in Table II.

5. Impact of altering reference: a cumulative study

The impact of altering the reference parameters a_m, b_m & c_m is inspected. Considering sets 2, 3 and 4, the transfer function of the reference model described in eqn. 2, is selected in a way that can respond rapidly to a step input (Astrom & Wittenmark 1994, Coman & Boldisor),

$$G_m(s) = \frac{2.6130}{s^2 + 2.28558s + 2.6130} \tag{25}$$

The reference system having 5% settling time of 3.5 sec, damping factor 0.707 with a percentage overshoot of 4.3%.

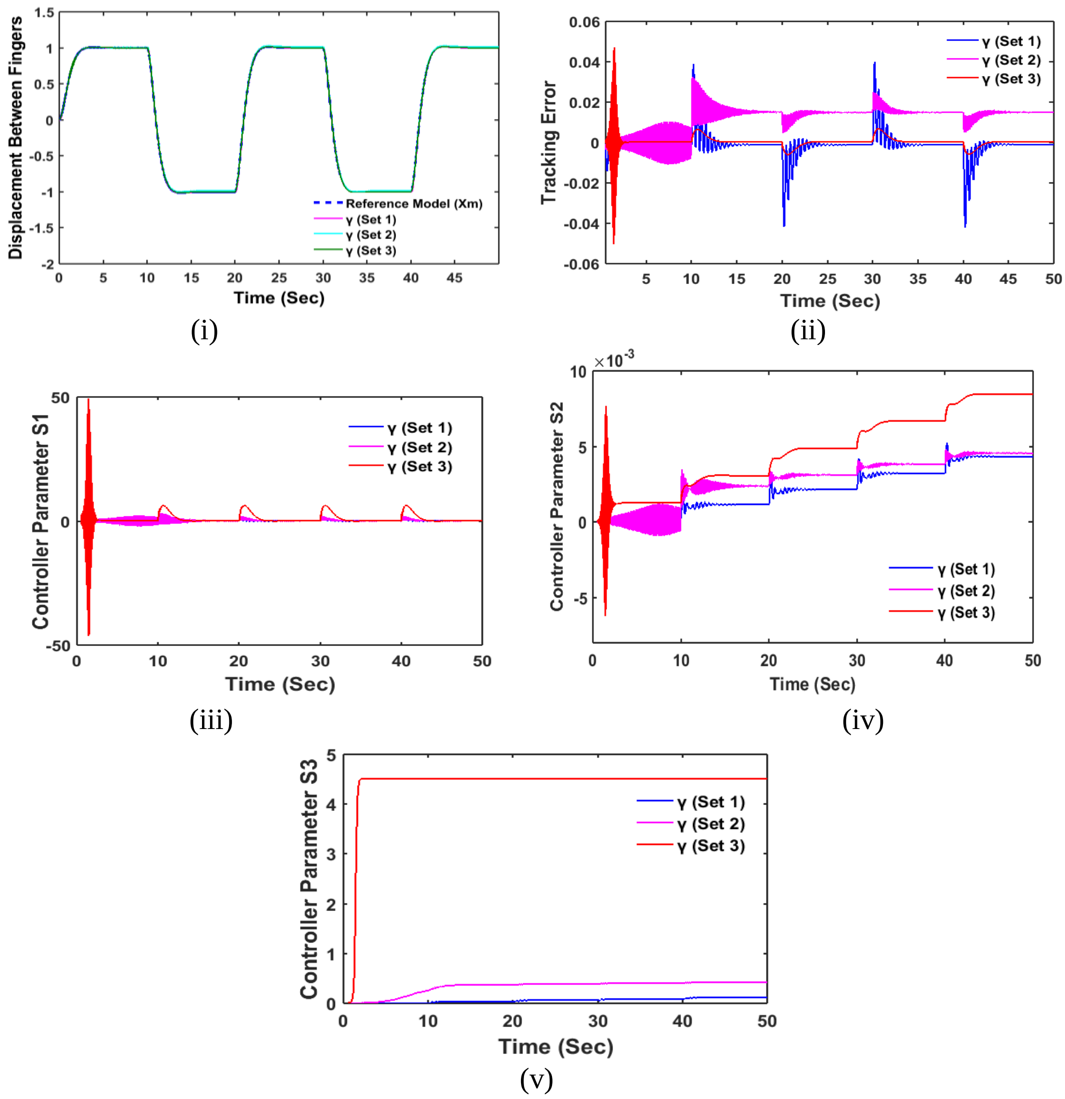


Figure 5. Lyapunov analysis, (i) time response corresponding to Square input, (ii)-(v) Tracking error corresponds to variations in gain (γ) of reference system and the process over time

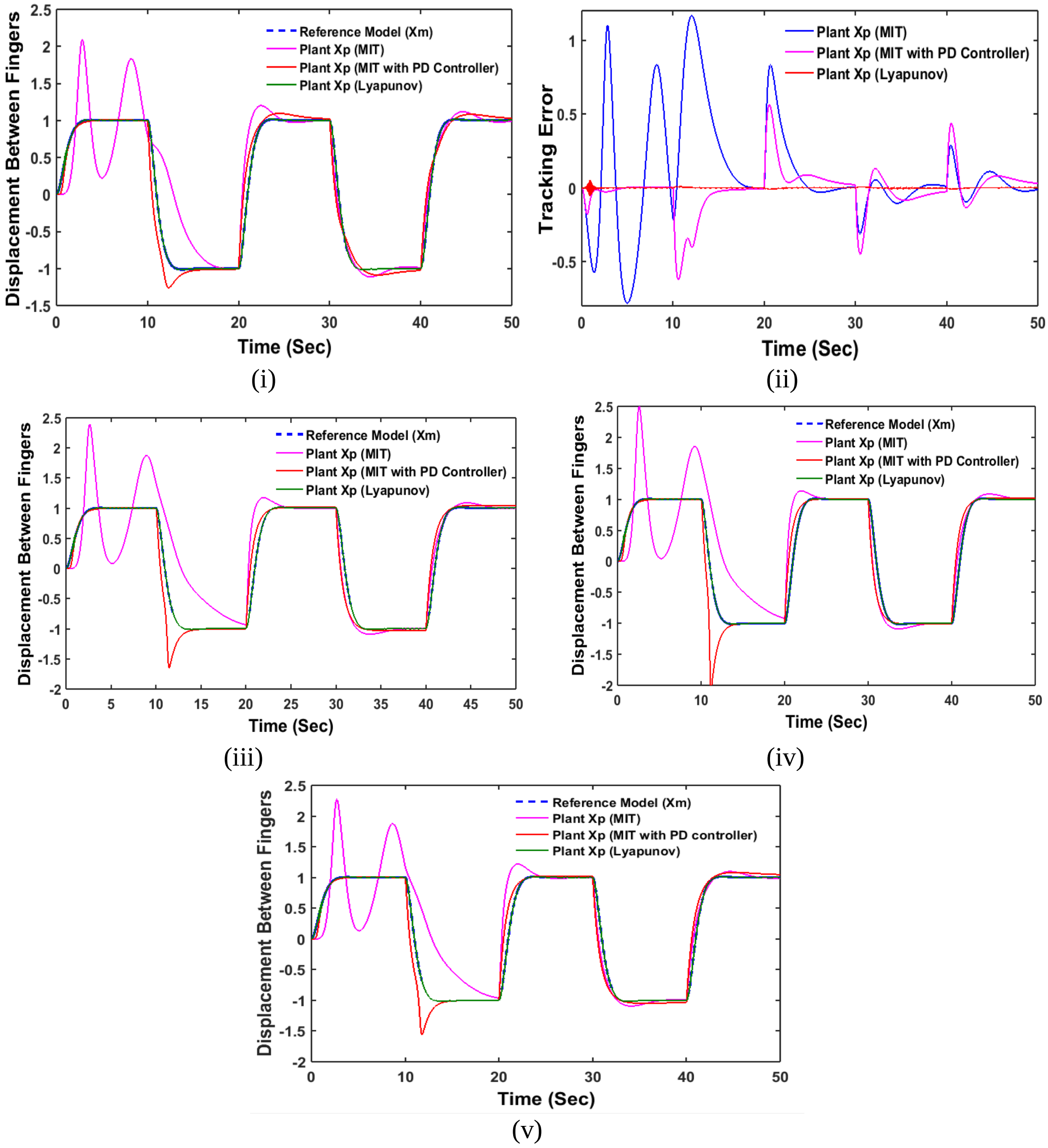


Figure 6. Comparative analysis, (i)-(ii) Time response for 1st set transfer function along with tracking error; (iii) Time Response for 2nd set transfer function; (iv) Time Response for 3rd set transfer function and (v) Time Response for 4th set transfer function

The approximate overshoot and 5% settling time for each three step spanning 10 seconds as seen in these simulations after an experiment with gain levels of 0.1, 2.4, and 5. Following are some of the conclusions drawn from these findings:

(1) Settling time reduces with the increase of gain value and takes less time for the plant's output to reach the desired reference model's response;

(2) With change of the gain, the frequency of oscillations increases and results in higher overshoot.

Higher gain levels result in a shorter settling period for the error signal, as evidenced by the time response. This comes at the cost of increased overshoot and may result in undesirable behaviour, such as the adaptation failing to converge. The MRAC without the PD controller has greater overshoot values and takes longer to stabilise the response. However, there is no severe overshoot and the response takes less time to stabilise.

Table II. Comparative analysis

Specifications	*Reference model (Xm)*	*Values of Gammas(γ)*								
		$\gamma = 0.1$			$\gamma = 2.4$			$\gamma = 5$		
		MIT	MIT-PD	Lyapu-nov	MIT	MIT-PD	Lyapu-nov	MIT	MIT-PD	Lyapu-nov
% Overshoot or Undershoot	0.63	94.2	-0.28	4.5	112	2.3	4.7	108	2.7	4.2
Peak time (Sec)	2.74	5.63	10	2.735	3.513	7.220	2.854	2.831	6.88	2.725
Settling Time (sec)	4.333	>10	10	4.267	>10	>10	4.17	>10	>10	4.282

6. Conclusions

Using MRAC methodologies, this paper proposes the design and analysis of an adaptive controller for a mechanical prosthetic arm. By assuring consistent output performance of a mechanical prosthetic limb within a range of adaption gain, the technique attempts to improve system transient responsiveness. For the analysis of one under-damped second order control system, the Gradient method MIT rule was introduced into the controller design of this second order system. The influence of adaption gain for different values of system parameters on overall system performance was investigated using computer-aided control system design (SACSD) tools such as MATLAB-SIMULINK. In the face of nonconformity of system parameters, Lyapunov stability analysis was utilised to ensure system global stability. Further, to evaluate the effect of the controller on the system's temporal response characteristics, such as minimum settling time and minimal percentage overshoot, a cumulative study on the influence of variation of modifying the reference model was conducted.

REFERENCE

1. Anita, R. & Prakash, R. (2010). *Robust Model Reference Adaptive PI Control*, J. Theor. Appl. Inf. Technol., vol. 14.
2. Astrom, K. J. & Wittenmark, B. (1994). *Adaptive control*; 2nd Edition: Prentice-Hall.
3. Coman, S. & Boldisor, C. R. (2014). *Adaptive PI controller design to control a mass-damper-spring process*. Bulletin of the Transilvania University of Braşov Series I Vol. 7 (56), No. 2.
4. Engeberg, E. D. (2013). *Human model reference adaptive control of a prosthetic hand*. Journal of Intel Robot Syst., doi: 10.1007/s10846-013-9815-9.
5. Kalman, R. E. (1958). *Design of a self-optimization control system rans*. ASME, vol. 80, 468-478.
6. MATLAB & Statistics Toolbox Release (2015a, 2015). *The Math Works, Inc., Natick, Massachusetts, United States. February. https://in.mathworks.com/help/simulink/ug/starting-simulink-software.html.*
7. Neogi, B., Islam Sanaul S. K., Chakraborty, P., Barui S., Das, A. (2016). *Mechanical Prosthetic Arm Control Model Using MIT Rule*. 4th International Conference on Advanced Computing, Networking, and Informatics, (ICACNI-2016).
8. Neogi, B., Ghosal, S., Ghosh, S., Bose T. K., Das. A. (2012). *Dynamic modeling and optimizations of mechanical prosthetic arm by simulation technique*. Recent Advances in Information Technology (RAIT), IEEE Xplore, pp. 883-888.
9. Oltean, S. E. & Morar, A. (2010). *Simulation of the local model reference adaptive control of the robotic arm with DC motor drive.* ACTA Electrotechnica, Vol. 51, No. 2, 114–118.

10. Rossini et al. (2010). *The Italian Tribune Newspaper*, July 21, 2010. http://theitaliantribune.com/?p=119.
11. Stellet, J. E. (2011). *Influence of Adaptation Gain and Reference Model Parameters on System Performance for Model Reference Adaptive Control*. World Academy of Science, Engineering and Technology 60, 1768–1773.
12. Sun J. (1993). *A modified model reference adaptive control scheme for improved transient performance*, IEEE Trans. Autom., Contr., vol. 38, no. 8, pp. 1255–1259.
13. Taifour Ali, Bashier Eisa & Omar Busat alzain Mohd (2012). *Adaptive PID Controller for Dc Motor Speed Control*. International Journal of Engineering Inventions, September 2012.
14. Tao, G. (2003). *Adaptive Control Design and Analysis*, S. Haykin, Ed. WileyInterscience.
15. Wise, K., Lavretsky, E. & Hovakimyan, N. (2006). *Adaptive control of flight: theory, applications, and open problems*. In American Control Conference.
16. Xiu-Chun, Zhao et al. (2012). *Model Reference Adaptive Control for Perforated Mill*. International Conference on Electronic & Mechanical Engineering and Information Technology, Dalian Nationalities University.

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