Fuzzy Sliding Mode with adaptive gain control for nonlinear MIMO systems

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Abstract: In this paper, a fuzzy sliding mode controller (FSMC) with adaptive gain is proposed for a class of MIMO nonlinear systems. The coupled system can be divided into two different subsystems, each of which can be separated into two sliding surfaces created by the state variables. The integration of these two sliding surfaces requires the introduction of an intermediary variable. In order to ensure that the control system is stable, the adaptive gain is derived using the Lyapunov method. The synthesized fuzzy sliding mode control with adaptive gain is applied to an inverted pendulum to illustrate the suggested control technique. The performance and robustness of the proposed controller are compared, terms of time, with those of provided by fuzzy sliding mode controller. This comparison reveals the superiority of the proposed method over the fuzzy sliding mode controller, in terms of the system stabilization and tracking of the trajectory.

Keywords: Fuzzy Controller, Sliding Mode Control, Adaptive Gain, Nonlinear Systems, Lyapunov Function.

1. Introduction

In general, designing a controller for a nonlinear system is a difficult task even when the dynamic model of the system is available. It becomes also more difficult especially when the dynamic model is unknown or poorly described. Moreover, the progress of research over the last two decades in synthetic methods has given rise to systematic approaches, providing thus high effective nonlinear control laws. Among them, it can be mentioned the input-output linearization approach based control analysis as well as the synthesis method for a large class of non-linear systems (Nayfeh, 2008).

However, these approaches can only be used for nonlinear systems where the dynamic model is well-known. To overcome this drawback, several adaptive control approaches have been introduced (Fradkov et al., 2013; Kokotović & Arcak, 2001). Among them, fuzzy adaptive control has been a considerable success (Yin et al., 2015; Sun et al., 2018). It should be noted that the main issue appeared in this approach is the possibility of dividing by zero, leading therefore to undefined control law. Accordingly, the inverse of the control gain is directly estimated to avoid this problem. In most fuzzy adaptive control approaches, a robustification step based sliding mode control law should be introduced (Sui & Zhao, 2022; Zhang et al., 2021).

Fuzzy system-based adaptive control methodologies have engaged great interest in practical control engineering and are emerging as potential strategies for controlling highly uncertain and nonlinear dynamical systems. In the last two decades, a number of adaptive fuzzy control schemes have been developed based on the universal approximation theorem (Van Kien et al., 2019; Sun et al., 2020) for a class of single-input single-output (SISO) nonlinear uncertain systems (Younsi et al., 2017; Hung et al., 2007). The multi-input multi-output (MIMO) nonlinear uncertain systems are investigated in (Yoshimura, 2015; Arefi & Jahed-Motlagh, 2013). The Lyapunov synthesis method is used to analyze the stability of such schemes (Cao et al., 2016). Direct and indirect schemes are conceptually two different approaches used to design a fuzzy adaptive control system in order to achieve control objectives.

In this paper, it is demonstrated that a significant class of fourth-order systems can be controlled without increasing the fuzzy rules, but by decreasing them to the minimum. The fuzzy

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controller can be constructed by human experts or by following some rules based on techniques derived in specialized literature (Lin et al., 2006; Ullah et al., 2015). The controller proposed in this paper transforms into a two-level sliding-mode controller for a specific class of fourth-order nonlinear systems if no language rules are provided.

The structure of this paper is as follows: Section 2 defines the terms and introduces the classic SMC design approach. Section 3 presents, the approach of designing fuzzy sliding mode control of coupled nonlinear systems that may be divided into two subsystems. Decoupled fuzzy sliding mode control with adaptive gain and stability analysis is shown in Section 4. In Section 5, the proposed controller is used to control highly nonlinear system inverted pendulum. Conclusions are finally summarized in the last section.

2. Sliding mode control

Considering a nonlinear system described by the following state representation (Ullah et al., 2015; Delavari et al., 2010):

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t) & \mathrm{i} = 1,\ldots,\mathrm{n\text{-}1} \\ \dot{x}_n(t) = f(X,t) + b(X,t)u(t) + d(t) \\ y(t) = x_1(t) \end{cases} \tag{1}$$

where: $X(t) = \left[x_1(t), x_2(t), \ldots, x_n(t)\right]^T$ is the state vector, $f(X,t)$ and $b(X,t)$ are nonlinear functions with $b(X,t) > 0$, $u(t)$ is the control, $d(t)$ is the disturbance considered to be bounded: $|d(t)| < D$.

The sliding surface is given by:

$$S(X,t) = \sum_{i=1}^{n-1} c_i x_i(t) + x_n(t) \tag{2}$$

where $c_i > 0$, $i = 1,\ldots,n-1$ are positive constants.

If a Lyapunov function of the following form is chosen:

$$V(X,t) = \frac{1}{2} S^2(X,t) \tag{3}$$

according to Lyapunov theorem, if $\dot{V}$ is negative, the state trajectory will be attracted to the sliding surface and commute around it to the point of equilibrium.

The derivative of the sliding surface is given by:

$$\dot{S}(X,t) = \sum_{i=1}^{n-1} c_i x_{i+1}(t) + f(X,t) + b(X,t)u(t) + d(t) \tag{4}$$

Multiplying equation (4) by S, it is obtained:

$$\dot{V}(X,t) = S(X,t)\dot{S}(X,t) = S(X,t)\left[\sum_{i=1}^{n-1} c_i x_{i+1} + f(X,t) + b(X,t)u(t) + d(t)\right] \tag{5}$$

The sliding mode control law is as follows (Amieur et al., 2014; Delavari et al., 2010):

$$u(t) = u_{eq}(t) - KSgn\left(S(X,t)\right), \quad K > \frac{D}{b(X,t)} \tag{6}$$

where: K is known control gain and $u_{eq}(t) = \dfrac{-\sum_{i=1}^{n-1} c_i x_{i+1} - f(X,t)}{b(X,t)}$ is the equivalent control.

By replacing $u(t)$ with its expression (6), the relation (5) becomes:

$$\begin{aligned} \dot{V}(X,t) &= S(X,t)\dot{S}(X,t) = S(X,t)\left[-KSgn\left(S(X,t)\right)b(X,t) + d(t)\right] \\ &= \left[-K\left|S(X,t)\right|b(X,t) + d(t)\right] \le -D\left|S(X,t)\right| + \left|d(t)\right|\left|S(X,t)\right| \le 0 \end{aligned} \tag{7}$$

$$\dot{V}(X,t) = S(X,t)\dot{S}(X,t) \le 0 \tag{8}$$

By applying the control equation (6), it results an unwanted oscillations (chattering) on the steady-state responses. The solution consists in replacing the function $Sgn(.)$ by the function $Sat(.)$ to avoid chattering problem and the temporal responses will be smoother:

$$u(t) = u_{eq}(t) - KSat\left(\frac{S(X,t)}{\Phi}\right), \quad K > \frac{D}{b(X,t)} \tag{9}$$

The control equation (9) can be depicted later, in Figure 3, and the saturation function is defined by:

$$Sat\left(\frac{S(X,t)}{\Phi}\right) = \begin{cases} Sgn\left(\dfrac{S(X,t)}{\Phi}\right) & if \quad \left|\dfrac{S(X,t)}{\Phi}\right| \ge 1 \\ \dfrac{S(X,t)}{\Phi} & if \quad \left|\dfrac{S(X,t)}{\Phi}\right| < 1 \end{cases}$$

, Φ : is the boundary layer of $S(X,t)$.

3. Fuzzy sliding mode control

The relation between the sliding surface $S(X,t)$ and the control $u(t)$ can be exploited to determine the basis of rules of a fuzzy controller allowing the asymptotic stability of the system (Lo & Kuo, 1998).

Suppose that the fuzzy controller is constructed from the following IF-THEN rules:

R^1: If S is NB Then u is BIGGER

R^2: If S is NM Then u is BIG

R^3: If S is ZR Then u is MEDIUM (10)

R^4: If S is PM Then u is SMALL

R^5: If S is PB Then u is SMALLER

where NB is negative big, NM is negative medium, ZR is zero, PM is positive medium, and PB is positive big. NB, NM, ..., SMALL, SMALLER are labels of fuzzy sets and their corresponding membership functions are depicted in Figure 1 and Figure 2, respectively (Lin et al., 2006; Lo & Kuo, 1998).

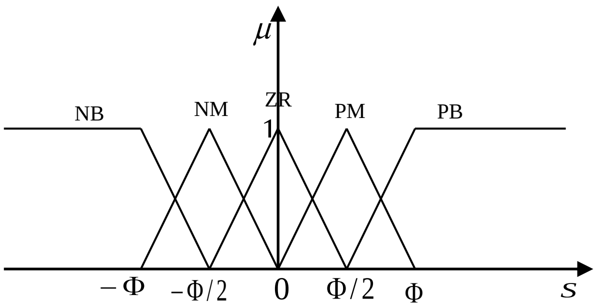

Figure 1. Membership functions for input *S*

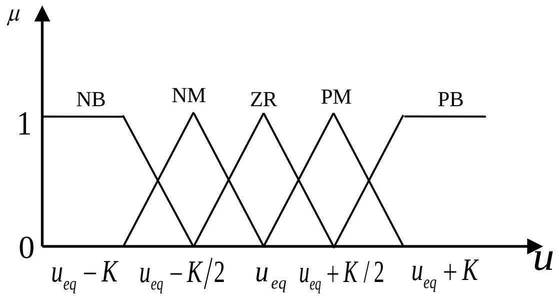

Figure 2. Membership functions for output *u*

By using the sup–min compositional rule of inference:

$$\mu_{\tilde{F}_x \circ R^i}(u) = \sup_{S \in X}\left[\min\left[\mu_{\tilde{F}_x}(S), \min\left[\mu_{\tilde{F}_S^i}(S), \mu_{\tilde{F}_u^i}(u)\right]\right]\right] \tag{11}$$

It can be further simplified by supposing that $\tilde{F}_x$ is a fuzzy singleton, i.e., only with its support $S = \alpha$, $\mu_{\tilde{F}_S^i}(S) = 1$ otherwise, $\mu_{\tilde{F}_S^i}(S) = 0$. Then (11) becomes:

$$\mu_{\tilde{F}_x \circ R}(u) = \min\left[\mu_{\tilde{F}_S^i}(\alpha), \mu_{\tilde{F}_u^i}(u)\right] \tag{12}$$

and the deduced membership function $\mu_{F_u^d}$ is:

$$\mu_{\tilde{F}_u^d}(u) = \max\left[\mu_{\tilde{F}_x \circ R^1}(u), \ldots, \mu_{\tilde{F}_x \circ R^5}(u)\right] \tag{13}$$

The crisp output u is obtained by the center-of-area defuzzifier:

$$u = \frac{\int_{-\frac{3}{2}}^{\frac{3}{2}} u\,\mu_{\tilde{F}_u^d}(u)\,du}{\int_{-\frac{3}{2}}^{\frac{3}{2}} \mu_{\tilde{F}_u^d}(u)\,du} \tag{14}$$

The result of the defuzzified output u for a fuzzy input S, has the following form:

$$u = u_{FSMC} = u_{eq} - KSat\left(\frac{S}{\Phi}\right) \tag{15}$$

The equation (15) is depicted in Figure 3.

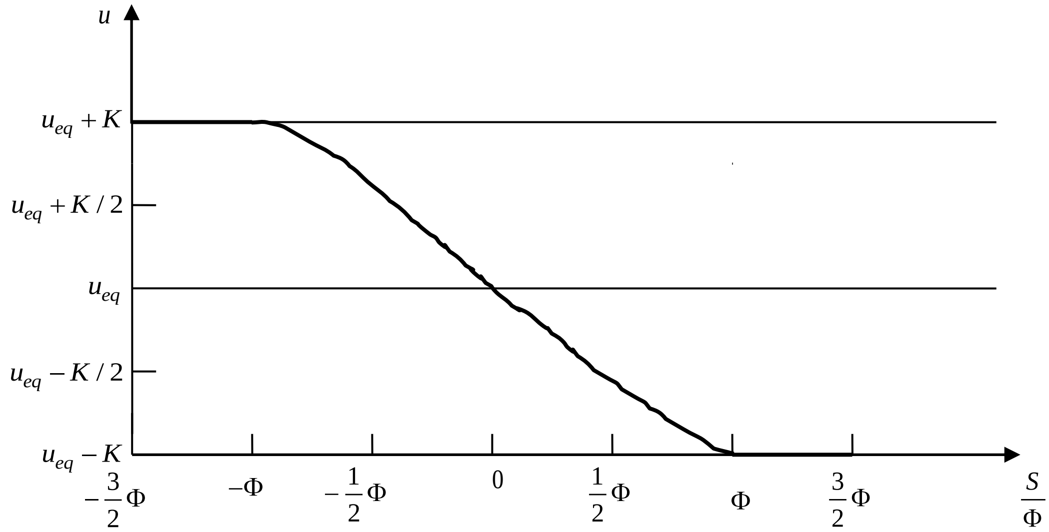

Figure 3. Result of defuzzification of a fuzzy controller *u*

4. Decoupled fuzzy sliding mode control

Consider a class of coupling nonlinear systems which can be divided into two subsystems as (Lin et al., 2006; Lo & Kuo, 1998):

$$\begin{cases} (A):\begin{cases}\dot{x}_1(t)=x_2(t)\\ \dot{x}_2(t)=f_1(X,t)+b_1(X,t)u(t)+d_1(t)\end{cases}\\ (B):\begin{cases}\dot{x}_3(t)=x_4(t)\\ \dot{x}_4(t)=f_2(X,t)+b_2(X,t)u(t)+d_2(t)\end{cases}\end{cases} \tag{16}$$

where $X(t)=\left[x_1(t),x_2(t),x_3(t),x_4(t)\right]^T$ is the state vector, $f_1(X,t)$, $f_2(X,t)$, $b_1(X,t)$ and $b_2(X,t)$ are nonlinear functions with $b_1(X,t)>\underline{b}_1>0$, $b_2(X,t)>\underline{b}_2>0$, respectively, $u\ \ t$ is the control and $d_1\ \ t$ and $d_2\ \ t$ are the disturbances supposed to be bounded:

$$\left|d_i(t)\right|<D_i,\quad i=1,2$$

Two sliding surfaces, namely $S_1(X,t)$ for the subsystem **A** and $S_2(X,t)$ for the subsystem **B**, are defined:

$$S_1(X,t)=c_1x_1(t)+x_2(t) \tag{17}$$

$$S_2(X,t)=c_2x_3(t)+x_4(t) \tag{18}$$

where c_1 and c_2 are positive constants.

From the sliding mode theory presented in the previous section, two control laws can be chosen: $u_1(t)$ for the subsystem **A** and $u_2(t)$ for the subsystem **B**, in the following form (Lin et al., 2006; Lo & Kuo, 1998):

$$u_1(t)=u_{1eq}(t)-K_1Sat\left(S_1(X,t)b_1(X,t)/\Phi_1\right) \tag{19}$$

with $K_1>\dfrac{D_1}{b_1(X,t)}$, $\left|\dfrac{S_1(X,t)b_1(X,t)}{\Phi_1}\right|<1$ and $u_{1eq}(t)=\dfrac{-c_1x_2(t)-f_1(X,t)}{b_1(X,t)}$

where K_1 is the control gain for $u_1(t)$ and Φ_1 is the boundary layer of $S_1(X,t)b_1(X,t)$.

$$u_2(t)=u_{2eq}(t)-K_2Sat\left(S_2(X,t)b_2(X,t)/\Phi_2\right) \tag{20}$$

with $K_2>\dfrac{D_2}{b_2(X,t)}$, $\left|\dfrac{S_2(X,t)b_2(X,t)}{\Phi_2}\right|<1$ and $u_{2eq}(t)=\dfrac{-c_2x_3(t)-f_2(X,t)}{b_2(X,t)}$

where K_2 is the control gain for $u_2(t)$ and Φ_2 is the boundary layer of $S_2(X,t)b_2(X,t)$.

Assuming that the essential objective is to stabilize the subsystem **A**, it is reasonable to consider the information coming from the subsystem **B** as secondary, and this secondary information must be taken into account by the subsystem **A**. Therefore, an intermediate variable z which represents this secondary information is incorporated into $S_1(X,t)$. The surface $S_1(X,t)$ takes the shape $c_1(x_1(t)-z)+x_2(t)$, which means that the main goal is changed to $x_1(t)=z$, $x_2(t)=0$, or z is a function of $S_2(X,t)$.

The expression $S_1(X,t)$ and $S_2(X,t)$ can be written as:

$$S_1(X,t)=c_1(x_1(t)-z)+x_2(t) \tag{21}$$

$$S_2(X,t)=c_2x_3(t)+x_4(t) \tag{22}$$

The control law becomes:

$$u_1(t)=u_{1eq}(t)-K_1Sat\left(S_1(X,t)b_1(X,t)/\Phi_1\right) \tag{23}$$

with

$$K_1\succ\frac{D_1}{b_1(X,t)}\ ,\ u_{1eq}(t)=\frac{-c_1x_2(t)-f_1(X,t)}{b_1(X,t)} \tag{24}$$

The value of the state z can be limited by posing

$$|z|\le z_U,\quad 0<z_U<1 \tag{25}$$

where z_U is the maximum value of $|z|$.

The variable z can be defined by:

$$z=Sat\left(\frac{S_2(X,t)}{\Phi_z}\right)z_U,\quad \left|\frac{S_2(X,t)}{\Phi_z}\right|<1\quad and\quad 0<z_U<1 \tag{26}$$

where Φ_z is the boundary layer of $S_2(X,t)$ which smooths the control and maintains the state of the system in this band. From equation (22), if $S_2(X,t)\neq 0$ then $z\neq 0$, if $S_2(X,t)\to 0$ then $z\to 0$, $x_1(t)\to 0$ and $S_1(X,t)\to 0$ so the purpose of the control can be completed (Delavari et al., 2010).

4.1. Fuzzy sliding mode control with adaptive gain

In the previous section it has been assumed that the gain K of the sliding mode control can be determined. However, in practice, there is no method for calculating this gain. To solve this problem, an adaptive control gain is used in this subsection.

Consider the following control law:

$$u(t) = u_{eq}(t) + u_n(t) \tag{27}$$

The equivalent control u_{eq} can be obtained from the time derivative of the surface $\dot{S}_1 = 0$.

$$\dot{S}_1(X,t) = c_1(\dot{x}_1(t) - \dot{z}) + \dot{x}_2(t) \tag{28}$$

$$\dot{S}_1(X,t) = c_1(x_2(t) - \dot{z}) + f_1(X,t) + b_1(X,t)u(t) \tag{29}$$

$$u_{eq}^*(t) = \frac{-c_1 x_2(t) + c_1\dot{z} - f_1(X,t)}{b_1(X,t)} \tag{30}$$

The variable z is not differentiable and $\dot{z}$ cannot be obtained. For this reason, the optimal equivalent control law $u_{eq}^*(t)$ is approached by the equivalent control $u_{eq}(t)$ given by:

$$u_{eq}(t) = \frac{-c_1 x_2(t) - f_1(X,t)}{b_1(X,t)} \tag{31}$$

and $k(t)$ is given by:

$$k(t) = u_{eq}(t) - u_{eq}^*(t) \quad \text{with} \quad 0 \le k(t) \le K \tag{32}$$

The uncertainty K limit is a positive constant. However, this uncertainty limit cannot be measured in practice (Amieur et al, 2014).

$\hat{K}(t)$ is the estimated value of K. The estimation error is considered as follows:

$$\tilde{K}\ t\ = K - \hat{K}\ t \tag{33}$$

The discontinuous control $u_n(t)$ whose purpose is to verify the conditions of attractiveness, is an adaptive sliding mode control term introduced to compensate the difference between the optimal equivalent control $u_{eq}^*(t)$ and the equivalent control $u_{eq}(t)$.

$$u_n(t) = -\hat{K}(t)Sign(S_1(X,t)b_1(X,t)) \tag{34}$$

To ensure the objectives of the control, the following adaptation law is taken into consideration:

$$\dot{\hat{K}}(t) = -\dot{\tilde{K}}(t) = \eta\left|S_1(X,t)b_1(X,t)\right| \quad \text{where} \quad \eta > 0 \tag{35}$$

4.2. Stability analysis

In order to demonstrate the stability of the system, the following Lyapunov candidate function is considered:

$$V(X,t) = \frac{1}{2}S_1^2(X,t) + \frac{1}{2\eta}\tilde{K}^2(t) \tag{36}$$

whose time derivative is:

$$\dot{V}(X,t) = S_1(X,t)\dot{S}_1(X,t) + \frac{1}{\eta}\tilde{K}(t)\dot{\tilde{K}}(t) \tag{37}$$

From (29) and (33), it is obtained:

$$\dot{V}(X,t) = S_1(X,t)\left(c_1 x_2(t) - c_1\dot{z} + f_1(X,t) + b_1(X,t)u(t)\right) + \frac{1}{\eta}\left(K - \hat{K}(t)\right)\dot{\hat{K}}(t) \tag{38}$$

By replacing $u(t)$ with its expression (27) and by using the adaptation law (35), the relation (38) becomes:

$$\begin{aligned}\dot{V}(X,t) &= S_1(X,t)\left(c_1 x_2(t) - c_1\dot{z} + f_1(X,t) + b(X,t)\left(u_{eq}(t) - u_{eq}^*(t) + u_{eq}^*(t) + u_n(t)\right)\right) \\ &-\left(K - \hat{K}(t)\right)\left|S_1(X,t)b_1(X,t)\right|\end{aligned} \tag{39}$$

From (30), it is obtained:

$$\dot{V}(X,t) = S_1(X,t)\left(b_1(X,t)\left(u_{eq}(t) - u_{eq}^*(t) + u_n(t)\right)\right) - \left(K - \hat{K}(t)\right)\left|S_1(X,t)b_1(X,t)\right| \tag{40}$$

From (32) and (34), it is obtained:

$$\dot{V}(X,t) = S_1(X,t)\left(b_1(X,t)\left(k(t) - \hat{K}Sign\left(S_1 b_1(X,t)\right)\right)\right) - \left(K - \hat{K}(t)\right)\left|S_1(X,t)b_1(X,t)\right| \tag{41}$$

$$S_1(X,t)b_1(X,t)\hat{K}Sign\left(S_1(X,t)b_1(X,t)\right) = \hat{K}(t)\left|S_1(X,t)b_1(X,t)\right| \tag{42}$$

$$\dot{V}(X,t) = S_1(X,t)b_1(X,t)k(t) - K\left|S_1(X,t)b_1(X,t)\right| \tag{43}$$

$$\dot{V}(X,t) \le 0.$$

5. Simulation and results

To demonstrate the effectiveness of the method inverted pendulum was used to ensure the control. The system consists of a movable cart in translation motion which supports a pendulum with free rotation, as shown in Figure 4.

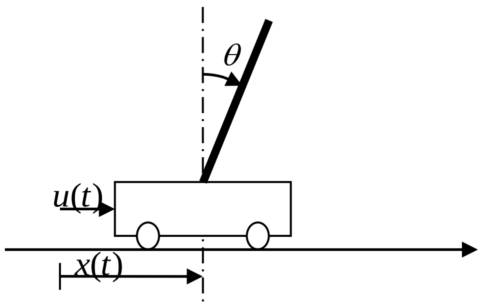

Figure 4. Structure of an inverted pendulum system

The motion can be described by the following differential equations (Jafary & Tabatabaei, 2022):

$$\begin{cases}\dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \dfrac{m_t g \sin x_1(t) - m_p L \sin x_1(t)\cos x_1(t) x_2^2(t)}{L.\left(\frac{4}{3}m_t - m_p \cos^2 x_1(t)\right)} + \dfrac{\cos x_1(t)}{L.\left(\frac{4}{3}m_t - m_p\cos^2 x_1(t)\right)}u(t) + d_1(t) \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = \dfrac{\frac{4}{3}m_p L x_2^2(t)\sin x_1(t) + m_p g \sin x_1(t)\cos x_1(t)}{\frac{4}{3}m_t - m_p\cos^2 x_1(t)} + \dfrac{4}{3.\left(\frac{4}{3}m_t - m_p\cos^2 x_1(t)\right)}u(t) + d_2(t)\end{cases} \tag{44}$$

where $x_1(t) = \theta(t)$, $x_2(t) = \dot{\theta}(t)$, $x_3(t) = x(t)$, $x_4(t) = \dot{x}(t)$, $u(t)$ is the force which moves the cart, $x(t)$ is the position of the cart and $\theta(t)$ is the angle of the pole.

$$m_p = 0.1kg\,,\; m_c = 1kg\,,\; m_t = m_c + m_p\,,\; L = 0.5m\,,\;\; g = 9.81m/s^2\,.$$

The simulation results are illustrated in Figures 5-16, for the following conditions: the initial conditions $x(0) = [-1.0, 0, -1.0, 0]^T$, the external disturbance $d_1(t) = d_2(t) = 0.5\sin(t)$ and for a controller by fuzzy sliding mode with adaptive gain whose parameters are the following: $c_1 = 5$, $c_2 = 0.5$, $\eta = 0.1$, $K(1) = 8$, $\Phi_1 = 5$, $\Phi_z = 8.5812$, $z_U = 0.9425$

5.1. Tracking rectangular signal reference position

Figures 5-8 illustrate the results obtained for tracking the following desired state vector: $\theta_d(t) = 0$, $\dot{\theta}_d(t) = 0$, $x_d(t) = \begin{cases} 1 & if \quad 15\sec \le t \le 30\sec \\ -1 & if \qquad else \end{cases}$ and $\dot{x}_d(t) = 0$

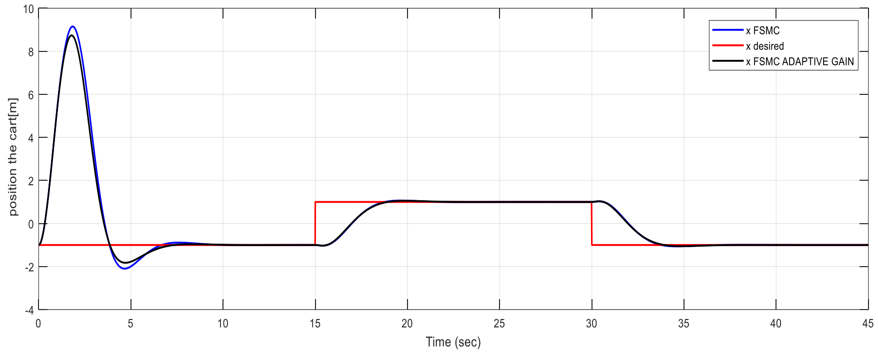

Figure 5. Position evolution of the cart $x(t)$ and $x_d(t)$

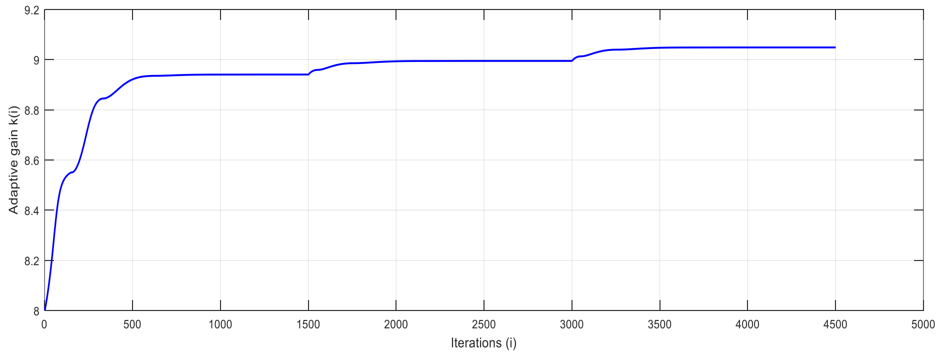

Figure 6. Evolution of the adaptive gain

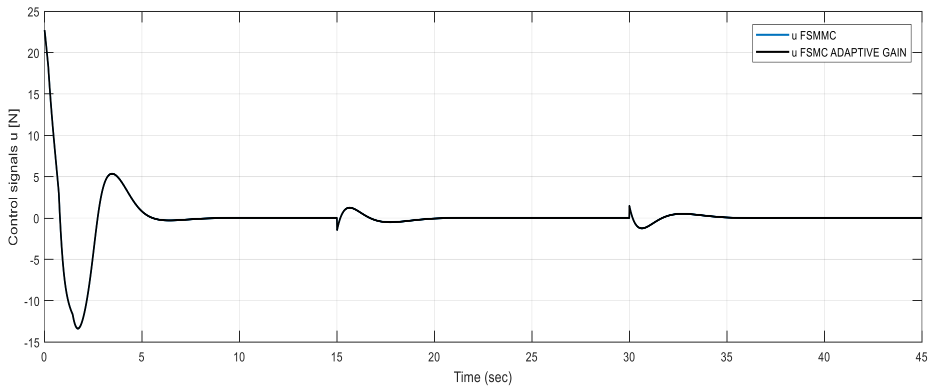

Figure 7. Control signals by $u(t)$ FSMC and $u(t)$ FSMC adaptive gain

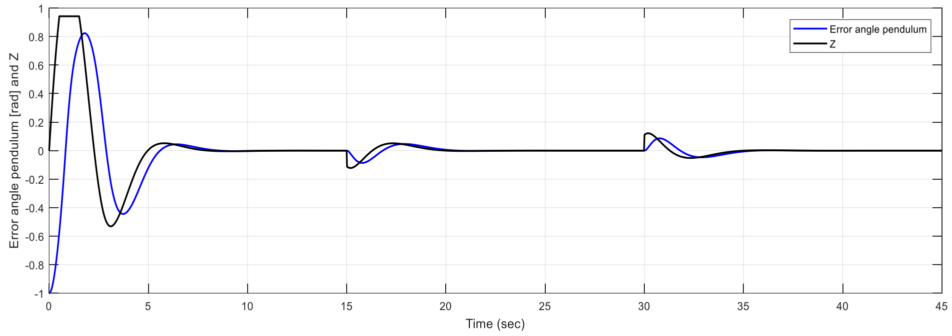

Figure 8. Evolution of error angle pendulum $\left[\theta(t)-\theta_d(t)\right]$ and variable z

5.2. Tracking sinusoidal signal reference position

Figures 9-12 illustrate the results obtained for tracking the following desired state vector:

$$\theta_d(t)=0,\ \dot{\theta}_d(t)=0,\ x_d(t)=\sin\left(\left(\frac{\pi}{30}\right)t\right) \text{ and } \dot{x}_d(t)=\left(\frac{\pi}{30}\right)\cos\left(\left(\frac{\pi}{30}\right)t\right)$$

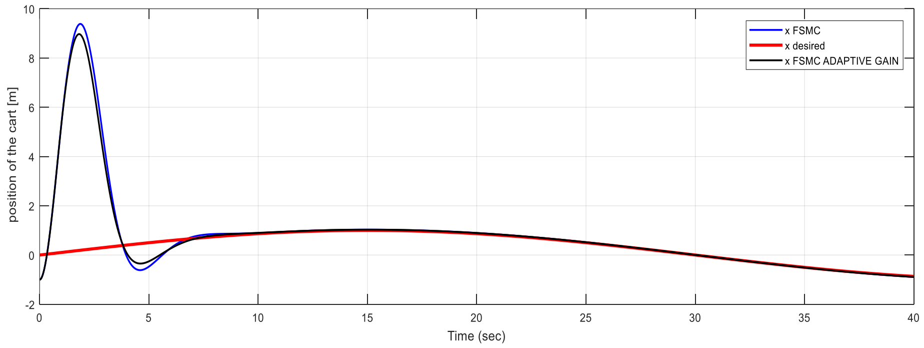

Figure 9. Position evolution of the cart $x(t)$ and $x_d(t)$

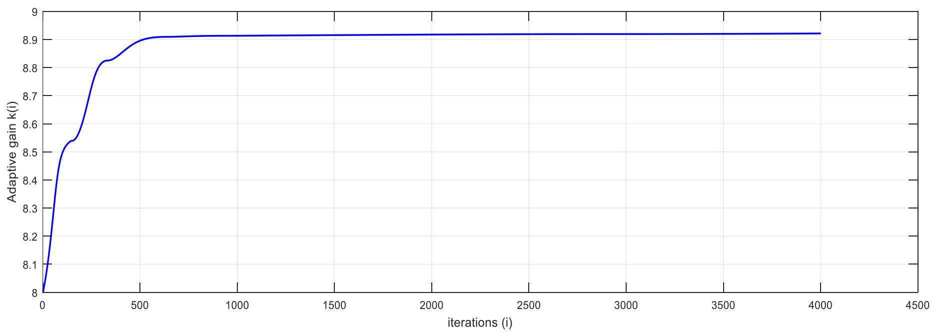

Figure 10. Evolution of the adaptive gain

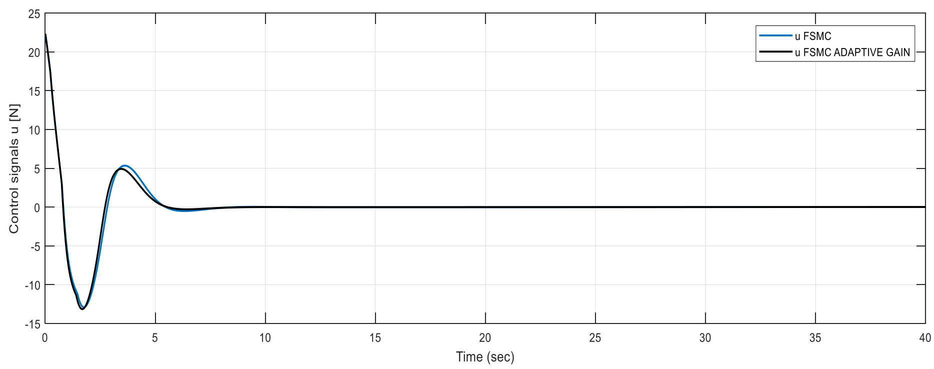

Figure 11. Control signals by $u(t)$ FSMC and $u(t)$ FSMC adaptive gain

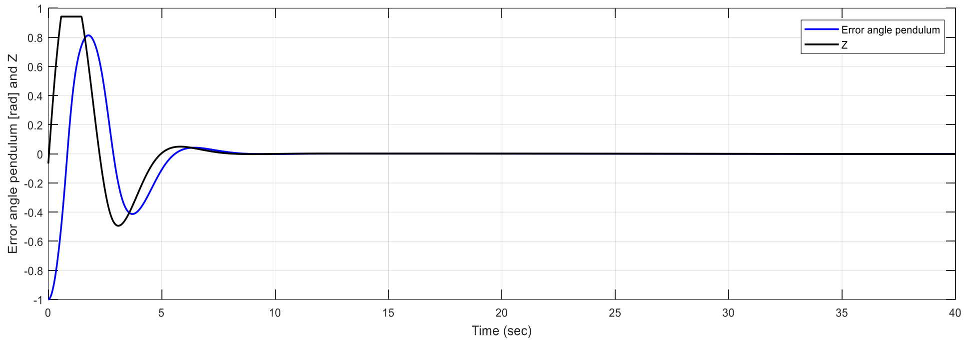

Figure 12. Evolution of error angle pendulum $\left[\theta(t)-\theta_d(t)\right]$ and variable z

5.3. Tracking ramp signal reference position

Figures 13-16 illustrate the results obtained for tracking the following desired state vector: $\theta_d(t)=0$, $\dot{\theta}_d(t)=0$, $x_d(t)=2t$ and $\dot{x}_d(t)=2$

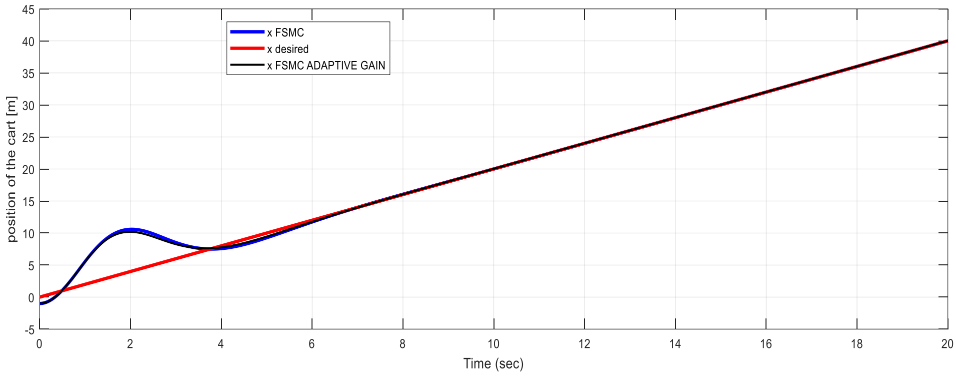

Figure 13. Position evolution of the cart $x(t)$ and $x_d(t)$

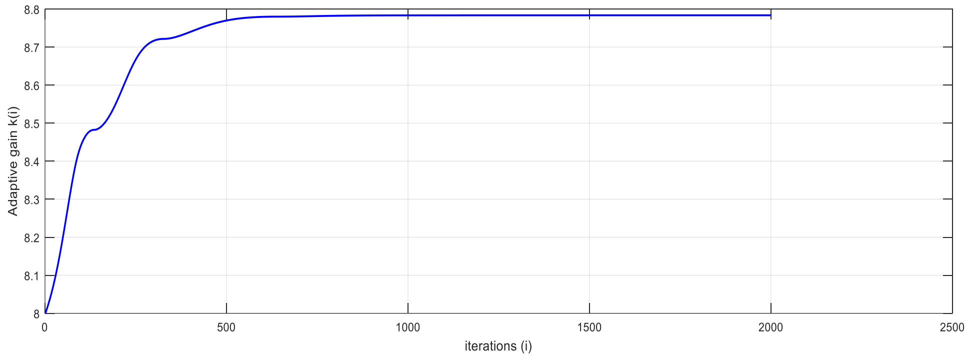

Figure 14. Evolution of the adaptive gain

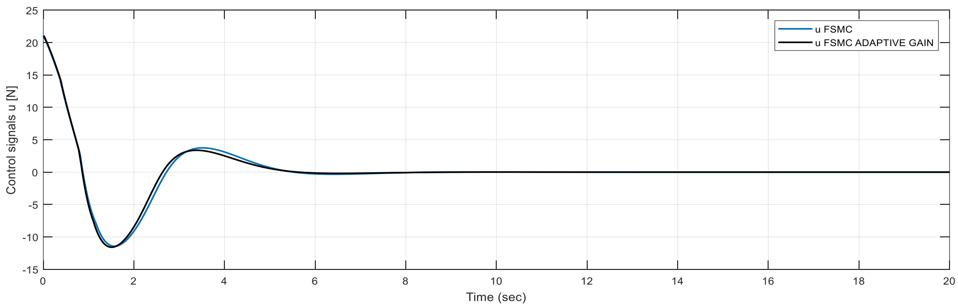

Figure 15. Control signals by $u(t)$ FSMC and $u(t)$ FSMC adaptive gain

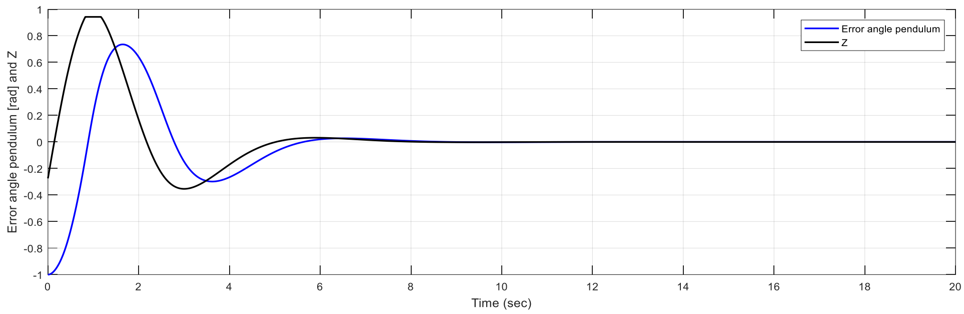

Figure 16. Evolution of error angle pendulum $\left[\theta(t)-\theta_d(t)\right]$ and variable z

The sliding mode control forces the system to follow the reference signal. This tracking is done by minimizing the tracking error, on the one hand, and by ensuring the stability of the system, on the other hand. The simulation results show the efficiency and performance of the fuzzy sliding mode control with adaptive gain. It can be seen that this controller has eliminated the chattering and ensured the smoothing of the control, the stabilization of the system and the tracking of the trajectory.

6. Conclusion

A class of coupling nonlinear systems is presented using fuzzy sliding mode control (FSMC). The idea of the control law is based on the following. First, the coupling system is divided into two independent subsystems, each with a unique control target that is represented in terms of a sliding surface. To include these sliding surfaces, an intermediary variable is added. Moreover, the Lyapunov approach is adopted in order to achieve the stability of the closed-loop system. Finally, the proposed method has been applied for examples of selected highly nonlinear systems that have been presented and the simulation results have validated the tracking of a reference trajectory. The simulation results demonstrate that the fuzzy sliding mode control with adaptive gain can achieve the desired performance.

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