Predictor-based event-triggered control for nonlinear cyber-physical systems under channel-induced delay

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Abstract: This article addresses the issue of designing a predictor-based event-triggered control strategy for cyber physical systems with channel induced delay over the controller – actuator portion of the communication channel. To deal with the conservatism posed on the system performance and stability by the channel delay a model-based predictor is first designed and a control policy is subsequently formulated using the predictor states. This predicted version of the control policy effectively mitigates the ill effects of the channel delay. To ensure the optimum utilization of the channel resources and to reduce the computational burden an event-triggered version of the control policy is devised. The event-triggered control proposed in this work ensures the Zeno free behavior of the system and the Lyapunov-function-based analytical developments reflect the uniform ultimate boundedness (UUB) of the state variables with the prescribed system performance. The validation of the effectiveness of the presented control scheme is illustrated with the help of numerical simulation.

Keywords: Cyber Physical Systems (CPS), Channel Delay, Model-Based Predictor, Event-Triggered Control.

1. Introduction

A cyber physical system (CPS) can be viewed as a synergistic aggregation of computing devices and physical systems interacting with each other via sensors and actuators over a communication channel. Thus, CPS is a feedback system engineered to steer some remotely operated physical system by leveraging the capabilities of the computer, network, sensor and actuator. Technical advancements witnessed in the computing and communication realms have enhanced the vitality of this concept by exploring its potential prospects and spreading it over a wide spectrum of application areas such as telerobotics, smart grids, unmanned vehicles, traffic management, industrial process control, mobile sensor networks, medical equipments and others. Several researchers have addressed the issues of CPS modelling and control.

Although the CPS has a plethora of promising prospects, it is associated with certain limitations and constraints posed by the environmental randomness, material characteristics and the openness of the architecture. The band width limitation, propagation and actuation delay, data loss, channel noise and cyber-attacks are some of the factors which may cause an impairment of the system performance (Lee, 2008; Alur, 2015; Zanero, 2017). This work addresses two crucial factors affecting the performance of networked systems, namely the channel resource limitation and the channel induced delay.

The network induced delay is one of the factors responsible for the degeneration of the system performance and it sometimes even leads to a system instability. It is therefore required to augment the baseline control scheme with a delay compensating mechanism to alleviate the effects of the delay (Jamaludin & Rohani, 2018; Wang et al., 2011). Various delay compensating techniques and analysis of their effectiveness in concern to networked induced delay have been cited in the literature (Angeli & Sontag, 1999; Ma et al., 2019; Zhu et al., 2023).

The model-based predictor techniques have been proved highly effective to compensate the effect of a network delay. In comparison to a delay model like pade approximations this technique relaxes the constraint of the upper bound on the delay size and thereby allows an accurate prediction of the system variables for any arbitrary delay values. The predictors are designed to predict the values of the state variables over a prediction horizon equal to a channel delay. The predicted states are used to formulate the control scheme. The control term formulated with the predictor states effectively results in a delay free feedback loop and thus mitigates the influences of the delay. The predictor based delay compensation technique relaxes the conservatism of the delay

https://doi.org/10.33436/v35i3y202505

dependency and has been proved highly effective in the case of an arbitrarily long delay (Tabuada 2007; Petit & Krstic, 2015; Vafaei & Yazdanpanah, 2017; Lee & Son, 2023). The event-triggered scheme has been proved highly effective to optimize the utilization of communication and computing resources. The event-triggering mechanism allows the control signal update and transmit only at some discrete instants which are governed by a predesigned event-triggering mechanism. These triggering rules are designed as to ensure the system performance within the desired limits of stability and performance. This control methodology seems highly effective from a resource utilization point of view as it uses computing and communication resources only at certain instants rather than in a continuous way. This scheme, instead of keeping a tight rein on the physical systems, tries to keep a balance between system performance and resource utilization (Cai et al., 2019; Xie, Ma & Xu, 2022).

The issue of the event-triggered control of the time delay system has been addressed by some researchers. An event triggered under the condition of a delayed channel often complicates the situation as extensively delayed control updates may not be able to preserve the system stability. This approach points towards the requirement of an accurate prediction model to predict the system states in synchronization with the channel delay (Al Issa & Kar, 2022a; Luo et al., 2023).

The following can be considered as the contribution of this paper:

- Designing of a Predictor-Based Event-Triggered Control Framework:

A model-based predictor is designed to precisely estimate the system states over a prediction horizon that corresponds to the delay imposed by the channel. This predictor effectively mitigates the negative impacts of communication latency by delivering dependable forecasts of future states. Utilizing these anticipated states, a policy for event-triggered control is developed to minimise superfluous control updates while guaranteeing prompt actuation.

- Delay Compensation and Stability Guarantees:

Rigid stability conditions for the proposed predictor are derived using a Lyapunov-Krasovskii-functional-approach. These standards ensure that the prediction error stays contained regardless of the channel-induced delay. In addition, with the suggested predictor-based control method, the Lyapunov-based stability requirements are set up to guarantee that the total closed-loop system is uniformly ultimate bounded (UUB).

- Avoidance of Zeno Behavior:

A formal explanation is given to show that the event-triggered mechanism's inter-execution time can be contained within a positively definite bound. This finding guarantees that the control method may be realistically implemented in real-time networked environments by effectively ruling out the possibility of Zeno behaviour.

This works presents a predictor-based event-triggered control scheme for a class of networked nonlinear systems with a channel induced delay. The following can be considered as the main contribution of this paper:

A model-based predictor is designed to predict the system states over a horizon equal to the channel delay. An event triggered control policy is subsequently formulated using the predicted states to compensate for the delay.

The Lyapunov-Krasovskii-functional-based stability norms have been established for the predictor, which ensures a bounded prediction error under the condition of arbitrary delay. Also, the Lyapunov-based stability conditions have been derived to ensure the uniform ultimate boundedness of the closed loop system under the action of a predictor-based control term.

The existence of a lower bound on the inter-execution time has been proved thereby ruling out the possibility of the Zeno behavior.

The rest of the paper is constituted as follows: Section 2 presents the system preliminaries. The predictor design and its stability issues are addressed in Section 3. Section 4 describes the design of an event triggered control scheme using predicted states. The validity aspects related to stability under the action of the control scheme and existence of the finite lower bound on the inter execution time are also addressed in this section. Section 5 demonstrates the numerical simulation study and the conclusion is presented in Section 6.

2. Preliminaries

2.1. Sytem Formation

Consider the following nonlinear system model as the mathematical representation of the physical system required to be steered over a communication channel. The physical system along with the communication channel and other components such as the controller, sensor and actuator constitute the cyber physical system (CPS). The channel attributes such as finite bandwidth and induced delay have been accounted during the controller design (Shu, Li & Xiang, 2024).

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ &\vdots \\ \dot{x}_n(t) &= f\left(x(t)\right) + u\left(t-\tau\right) \\ y\left(t\right) &= x_1\left(t\right) \end{aligned} \tag{1}$$

where $x = \left[x_1 \quad \dots \quad x_n\right]^T \in \Re^n; y \in \Re$, and $u(t-\tau) \in \Re$ denote the system states, output and control input delayed by τ instants. The term $f(x): R^n \to R$ represents the system nonlinearity and $\tau \in R^+$ is the delay induced between the controller and actuator.

To facilitate the subsequent developments, the nonlinear system (1) is expressed as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\left(f\left(x(t)\right) + u\left(t-\tau\right)\right) \\ y\left(t\right) &= Cx\left(t\right) \end{aligned} \tag{2}$$

where $A = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{nxn}; B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{nx1}; C = \left[c_1 \quad c_2 \quad \cdots \quad c_n\right]_{1xn}$ (3)

The objective of this work is to develop an event-triggered control policy for the system (1) considered under the condition of a cyber transmission with channel delay and finite bandwidth. The control term is required to steer the system output towards the desired trajectory $y_d(t)$ and the tracking error converges to the small neighborhood of its origin.

For circumventing the condition of the finite channel bandwidth, the event-triggered mechanism has been proved highly promising. This mechanism requires a sequence of triggering instants $\{t_k\}_{k=0}^{\infty}$ with $t_0 = 0$ and $\lim_{x\to\infty} t_k \to \infty$ along with a piecewise constant control term defined as

$$u(t) - u(t_k); t \in [t_k, t_{k+1}) \tag{4}$$

The control term so framed subsequently undergoes a channel delay and so the system is actuated by $u(t-\tau)$ instead of $u(t)$ resulting in system detuning. To alleviate the bad effects of the channel delay, the predictor-based controller design has been proved to be a promising technique. In this methodology, the control policy framed by using the predicted values of the system states and the feedback system performs like a delay free system.

The following assumptions are taken for the controller design:

Assumption 1: The nonlinear function $f(x(t))$ satisfies the condition of linear growth. Over a compact set $\Omega_x \subset R^n$ the condition can be expressed as (Khalil & Grizzle, 2002):

$$\|f(x(t)) - f(x(t_k))\| \le L\|x(t) - x(t_k)\| \tag{5}$$

where $L \succ 0$ is the Lipschitz constant.

Assumption 2: The desired tracking trajectory $y_d(t) \in R$ and its derivatives are bounded and known $\hat{y}_d(t) = \{y_d(t), \dot{y}_d(t), \ddot{y}(t), \cdots\} \in L_\infty$.

Assumption 3: The system (1) does not exhibit finite escape time for $t \in [0, \tau]$ for any initial condition and bounded input.

Assumption 4: The control input $\{u(t) | t \in [-\tau, 0]\}$ is bounded and known.

Assumption 5: There exists a predictor gain matrix m such that the matrix $(A - m)$ is Hurwitz stable. Also, there exists an arbitrary positive definite matrix Q such that a positive definite matrix P can be considered as the unique solution of the following equation:

$$(A - m)^T P + P(A - m) = -Q \tag{6}$$

Assumption 6: There exists a controller gain matrix K such that the matrix $(A - BK)$ is Hurwitz stable and under the condition of existence of the arbitrary positive definite matrix $Q_{1,} P_1$ which is a positive definite matrix can be taken as the unique solution of the equation

$$(A - BK)^T P_1 + P_1(A - BK) = -Q \tag{7}$$

Lemma 1. For any two matrices P and Q, following inequality holds

$$P^T Q + Q^T P \le \varepsilon P^T P + \frac{1}{\varepsilon} Q^T Q \tag{8}$$

where $\varepsilon \succ 0$

3. Predictor Design and Analysis

This section describes the development of the model-based predictors. The predictor states will be thereafter used for formulating the feedback law.

3.1. Predictor Design

In the context of the delay induced during the signal propagation from the controller to the actuator, a state predictor-based delay compensating strategy is formulated. A system model-based predictor for predicting the future values of the system states can be modelled as (Krstic, 2009):

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\left(f\left(\hat{x}(t)\right) + u(t)\right) + me_p(t) \tag{9}$$

where $\hat{x} = [\hat{x}_1, \cdots, \hat{x}_n]^T \in R^n$ are the predictor states. These states are the τ instants ahead predicted estimates of the system states, $e_p(t)$ is the prediction error defined as

$$e_p(t) = x(t) - \hat{x}(t - \tau) \tag{10}$$

and m is the gain matrix.

The term $me_p(t)$ is included to facilitate the rapid convergence of the prediction error to the close neighborhood of its origin.

Under the condition of event triggering, (9) can be expressed as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\left(f\left(\hat{x}(t)\right) + u(t)\right) + m\hat{e}_p(t) \tag{11}$$

where $\hat{e}_p(t)$ is a piecewise constant term which undergoes updating at the event triggering instants $\{t_k\}_{k=0}^{\infty}$. The term is defined as

$$\hat{e}_p(t) = e_p(t_k) = x(t_k) - \hat{x}(t_k - \tau); t \in [t_k, t_{k+1}) \tag{12}$$

The differentiation of (10) and the subsequent substitution of (2) and (9) results in the following dynamics:

$$\dot{e}_p(t) = \dot{x}(t) - \dot{\hat{x}}(t-\tau) = (A-m)e_p(t) + B(f(x(t)) - f(\hat{x}(t-\tau))) + m(e_p(t) - \hat{e}_p(t-\tau)) \tag{13}$$

To ensure the accurate prediction of the system states it is necessary that the prediction error dynamics (13) should converge to a residual set containing the origin. The convergence analysis of the prediction error is presented in the next subsection.

3.2. Convergence Analysis

Consider the following Lyapunov-Krasovskii functional (Al Issa & Kar, 2022b)

$$V_1 = \frac{1}{2}e_p(t)^T P e_p(t) + \int_{t-\tau}^{t} W(\theta)d\theta \tag{14}$$

where $e_p(t)$ (10) is the prediction error, P is a positive definite symmetric matrix and $W(\theta)$ is a positive definite function which will be defined later.

The differentiation of (14) yields the following expression

$$\dot{V}_1 = \frac{1}{2}(\dot{e}_p(t)^T P e_p(t) + e_p(t)^T P \dot{e}_p(t)) + W(t) - W(t-\tau) \tag{15}$$

The substitution of dynamics (13) results in

$$\dot{V}_1 = \frac{1}{2}\begin{pmatrix} \left\{(A-m)e_p(t) + B(f(x(t)) - f(\hat{x}(t-\tau))) + m(e_p(t) - \hat{e}_p(t-\tau))\right\}^T Pe_p(t) + \\ e_p(t)^T P\left\{(A-m)e_p(t) + B(f(x(t)) - f(\hat{x}(t-\tau))) + m(e_p(t) - \hat{e}_p(t-\tau))\right\} \\ +W(t) - W(t-\tau) \end{pmatrix} \tag{16}$$

Rearranging the equation and applying Lemma 1 and the conditions stated in assumptions 1-6:

$$\dot{V}_1 \le \frac{1}{2}\begin{pmatrix} e_p(t)^T\left\{(A-m)^T P + P(A-m)\right\}e_p(t) + e_p(t)^T m^T P e_p(t) + e_p(t)^T P m e_p(t) + \\ \frac{1}{\varepsilon_1}\left\{(f(x(t)) - f(\hat{x}(t-\tau))^T B^T B(f(x(t)) - f(\hat{x}(t-\tau))\right\} + \varepsilon_1 e_p(t)^T PPe_p(t) \\ +\frac{1}{\varepsilon_2}\left\{\hat{e}_p(t-\tau)^T m^T m\hat{e}_p(t-\tau)\right\} + \varepsilon_2 e_p(t)^T PPe_p(t) + W(t) - W(t-\tau) \end{pmatrix}$$

$$\leq \frac{1}{2}\left(\begin{array}{l} e_p(t)^T\left\{(A-m)^T P + P(A-m)\right\} e_p(t) + e_p(t)^T m^T P e_p(t) + e_p(t)^T P m e_p(t) + \\ \frac{1}{\varepsilon_1}\left\{(x(t)-\hat{x}(t-\tau))^T L^T L(x(t)-\hat{x}(t-\tau))\right\} + \varepsilon_1 e_p(t)^T P P e_p(t) \\ +\frac{1}{\varepsilon_2}\left\{\hat{e}_p(t-\tau)^T m^T m \hat{e}_p(t-\tau)\right\} + \varepsilon_2 e_p(t)^T P P e_p(t) + W(t) - W(t-\tau) \end{array}\right) \quad (17)$$

$$\leq \frac{1}{2}\left(\begin{array}{l} e_p(t)^T\left\{-Q + \frac{1}{\varepsilon_1} L^2 I + \varepsilon_1 P^2 + \varepsilon_2 P^2 + m^T P + P m\right\} e_p(t) + \\ +\frac{1}{\varepsilon_2}\left\{\hat{e}_p(t-\tau)^T m^T m \hat{e}_p(t-\tau)\right\} + W(t) - W(t-\tau) \end{array}\right)$$

Selecting the term $W(t)$ as

$$W(t) = \frac{1}{\varepsilon_2}\hat{e}_p(t)^T m^T m \hat{e}_p(t) \quad (18)$$

the substitution of $W(t)$ in (17) results in

$$\dot{V}_1 \leq \frac{1}{2}\left(e_p(t)^T \left\{-Q + \frac{1}{\varepsilon_1} L^2 I + \varepsilon_1 P^2 + \varepsilon_2 P^2 + m^T P + P m\right\} e_p(t) + \frac{1}{\varepsilon_2}\hat{e}_p(t)^T m^T m \hat{e}_p(t)\right) \quad (19)$$

At the triggering instances state variables are on the verge of violating the triggering rules which are defined considering the limits of stability. Under this condition it is reasonable to assume that there exists a finite value $\upsilon \in \Re^+$ such that, if the system undergoes legitimate triggering following the prescribed triggering rule, then following condition is valid

$$\hat{e}_p(t)^T m^T m \hat{e}_p(t) \leq \upsilon \quad (20)$$

Then the inequality (19) results in

$$\dot{V}_1 \leq \frac{1}{2}\left(\left\| e_p(t)\right\|^2 \left\{-\lambda_{\min}(Q) + \frac{1}{\varepsilon_1} L^2 + (\varepsilon_1+\varepsilon_2)\lambda_{\max}(P^2) + \lambda_{\max}(m^T P) + \lambda_{\max}(Pm)\right\} + \frac{1}{\varepsilon_2}\upsilon\right) \quad (21)$$

where $\lambda_{\min}(.)$ and $\lambda_{\max}(.)$ are minimum and maximum eigenvalues of the matrices under consideration. Thus $\dot{V}_1$ is negative outside a compact set defined

$$\Omega_{e_p} = \left\{ e_p(t) \middle| \left\| e_p(t)\right\| \leq \sqrt{\frac{\frac{1}{\varepsilon_2}\upsilon}{\left\{\lambda_{\min}(Q) - \frac{1}{\varepsilon_1} L^2 - (\varepsilon_1+\varepsilon_2)\lambda_{\max}(P^2) - \lambda_{\max}(m^T P) - \lambda_{\max}(Pm)\right\}}} \right\} \quad (22)$$

With an appropriate selection of parameters, the set can be reduced to an arbitrary small region.

The next section describes the design of the event triggered control policy using the observer state estimates and analysis of the closed loop system performance.

4. Event trigered control policy

This section describes the development of the event triggered control policy using the predicted values of the state variables.

4.1. Event triggered control scheme

For the prediction model (9) and the predicted state variables $\hat{x} = \left[\hat{x}_1 \quad \dots \quad \hat{x}_n\right]^T$ with the desired trajectory $y_d(t)$ satisfying assumption 2, the tracking error vector can be defined as

$$e_d = \left[e_{d1} \quad \dots \quad e_{dn}\right]^T \tag{23}$$

Where

$$\begin{aligned} e_{d1} &= \hat{x}_1 - y_d \\ &\vdots \\ e_{dn} &= \hat{x}_n - \overset{n-1}{y}_d \end{aligned}$$

The differentiation of the error vector (23) results in the following expression

$$\dot{e}_d(t) = Ae_d(t) + B\left\{ f(\hat{x}(t)) + u(t) - \overset{n}{y}_d(t) \right\} + m\hat{e}_p(t) \tag{24}$$

For the predictor dynamics (24), a nominal version of the control term can be defined as

$$u(t) = -Ke_d(t) - f(\hat{x}(t)) + \overset{n}{y}_d(t) \tag{25}$$

where $K = [k_1 \quad k_2 \quad \cdots \quad k_{n-1} \quad 1]$ is the gain vector with $k_i \prec 0; i = 1, 2, \cdots, n-1$.

The control term required to be designed here is an event trigger version of (25). As per the definition of the event triggering, the control term generated should be a piecewise constant signal which undergoes updates only at some discrete triggering instants $\{t_k\}_{k=0}^{\infty}$ controlled by some triggering rules. The event triggered control policy is therefore (3)

$$u(t) = u(t_k); t \in [t_k, t_{k+1}) \tag{26}$$

with $u(t_k) = -Ke_d(t_k) - f(\hat{x}(t_k)) + \overset{n}{y}_d(t_k)$

For controlling the triggering instances, the following triggering rule is defined as

$$(\hat{x}(t) - \hat{x}(t_k))^T (\hat{x}(t) - \hat{x}(t_k)) \le m_1 \tag{27}$$

where m_1 is a positive constant reflecting the limit of the permissible deviations of the state variables from their values at previous triggering instant. The triggering takes place whenever the deviation exceeds the limit, and the stated condition gets violated thus a triggering instant can be defined as

$$t_{k+1} = \inf\left\{ t \ge t_k; (\hat{x}(t) - \hat{x}(t_k))^T (\hat{x}(t) - \hat{x}(t_k)) \ge m_1 \right\} \tag{28}$$

The next subsection details the convergence analysis with the event-triggered control term.

4.2. Convergence analysis

This subsection analyzes the boundedness of the closed loop signals under the action of the event triggered control schemes (3), (24), (26) and (28).

The substitution of the control term in (24) results in the following dynamics

$$\dot{e}_d(t) = (A - BK)e_d(t) + B\left\{ f(\hat{x}(t)) - f(\hat{x}(t_k)) + \overset{n}{y}_d(t_k) - \overset{n}{y}_d(t) + Ke_d(t) - Ke_d(t_k) \right\} + m\hat{e}_p(t) \tag{29}$$

For the analytic validation, consider a Lyapunov function of the form

$$V_2 = \frac{1}{2} e_d(t)^T P_1 e_d(t) \tag{30}$$

Here p_1 is the positive definite symmetric matrix. Differentiating (30) along the system trajectories

$$\dot{V}_2 = \frac{1}{2}\left\{ \dot{e}_d(t)^T P_1 e_d(t) + e_d(t)^T P_1 \dot{e}_d(t) \right\} \tag{31}$$

The substitution of (29) in the above expression results in

$$\dot{V}_2 = \frac{1}{2}\left\{ \begin{array}{l} \left\{ \begin{array}{l} (A-BK)e_d(t) + m\hat{e}_p(t) \\ +B\left\{ f(\hat{x}(t)) - f(\hat{x}(t_k)) + \overset{n}{y}_d(t_k) - \overset{n}{y}_d(t) + Ke_d(t) - Ke_d(t_k) \right\} \end{array} \right\}^T P_1 e_d(t) \\ +e_d(t)^T P_1 \left\{ \begin{array}{l} (A-BK)e_d(t) + m\hat{e}_p(t) \\ +B\left\{ f(\hat{x}(t)) - f(\hat{x}(t_k)) + \overset{n}{y}_d(t_k) - \overset{n}{y}_d(t) + Ke_d(t) - Ke_d(t_k) \right\} \end{array} \right\} \end{array} \right\}$$

Under the conditions stated in Lemma 1 and the assumptions 1-6, the subsequent mathematical developments are:

$$\dot{V}_2 = \frac{1}{2}\left\{ \begin{array}{l} e_d(t)^T((A-BK)^T P_1 + P_1(A-BK))e_d(t) + (\nu_1+\nu_2+\nu_3+\nu_4)e_d(t)^T P_1 P_1 e_d(t) + \\ \frac{1}{\nu_1}\hat{e}_p(t)^T mm\hat{e}_p(t) + \frac{1}{\nu_2}\left\|(f(\hat{x}(t)) - f(\hat{x}(t_k)))\right\|^2 + \frac{2}{\nu_3}\left\|K^T K\right\|_F \left\|(e_d(t) - e_d(t_k))\right\|^2 \\ +\frac{1}{\nu_4}\left\|(\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t))\right\|^2 \end{array} \right\}$$

$$\dot{V}_2 = \frac{1}{2}\left\{ \begin{array}{l} e_d(t)^T(-Q_1 + (\nu_1+\nu_2+\nu_3+\nu_4)P_1^2)e_d(t) + \frac{1}{\nu_1}\upsilon + \frac{1}{\nu_2}L^2\left\|\hat{x}(t) - \hat{x}(t_k)\right\|^2 \\ +\frac{1}{\nu_4}\left\|(\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t))\right\|^2 + \frac{2}{\nu_3}\left\|K^T K\right\|_F (\left\|\hat{x}(t) - \hat{x}(t_k)\right\|^2 + \left\|\hat{y}_d(t) - \hat{y}_d(t_k)\right\|^2) \end{array} \right\} \tag{32}$$

where $\|.\|_F$ is the Frobenius norm of the matrix under consideration. Triggering condition (27) and boundedness of desired trajectory allows the following development

$$\begin{array}{l} \dot{V}_2 = \frac{1}{2}\left\{ e_d(t)^T(-Q_1 + (\nu_1+\nu_2+\nu_3+\nu_4)P_1^2)e_d(t) + \frac{1}{\nu_1}\upsilon + (\frac{1}{\nu_2}L^2 + \frac{1}{\nu_3}\left\|K^T K\right\|_F)m_1 + \rho \right\} \\ \leq \frac{1}{2}\left\{ \begin{array}{l} \left\|e_d(t)\right\|^2(-\lambda_{\min}(Q_1) + (\nu_1+\nu_2+\nu_3+\nu_4)\lambda_{\max}(P_1^2)) + \frac{1}{\nu_1}\upsilon + \\ (\frac{1}{\nu_2}L^2 + \frac{2}{\nu_3}\left\|K^T K\right\|_F)m_1 + \rho \end{array} \right\} \end{array} \tag{33}$$

Where positive constant ρ is defined as

$$\frac{1}{\nu_4}\left\|(\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t))\right\|^2 + \frac{2}{\nu_3}\left\|K^T K\right\|_F \left\|\hat{y}_d(t) - \hat{y}_d(t_k)\right\|^2 \leq \rho \tag{34}$$

The equation (33) allows to define the following convergence set

$$\Omega_{e_d} = \frac{1}{2}\left\{ e_d(t) \middle| \left\| e_d(t) \right\| \le \sqrt{\frac{\frac{1}{\nu_1}\upsilon + (\frac{1}{\nu_2}L^2 + \frac{2}{\nu_3}\left\| K^T K \right\|_F)m_1 + \rho}{\lambda_{\min}(Q_1) - (\nu_1 + \nu_2 + \nu_3 + \nu_4)\lambda_{\max}(P_1^2)}} \right\} \tag{35}$$

Therefore, the term $\dot{V}_2$ is negative when $e_d(t)$ is outside the residual set parameters. By properly selecting the value of the parameters $\nu_1, \nu_2, \nu_3, \nu_4$, the size of the set Ω_{e_d} can be made arbitrarily small. Lower values of these parameters are advised as they will reduce the denominator term which has a profound effect.

From (33) $\quad (\nu_1 + \nu_2 + \nu_3 + \nu_4) < \dfrac{\lambda_{\min}(Q_1)}{\lambda_{\max}(P_1^2)}$

and by using it following the design inequalities, optimal values of the parameters can be obtained,

$$0 < \nu_1 \le \frac{1}{2}; 0 < \nu_3 \le \frac{1}{2}; 0 < \nu_4 < \frac{\max \left\| (\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)) \right\|^2}{10}$$

Thus, for the networked dynamical system (2) with a predictor model (9) and control policy (26), the prediction and tracking error converges to the residual sets (22) and (35) and all the closed loop signals are uniformly ultimately bounded.

The next section examines the compatibility of the event-triggering scheme under the constraints imposed by issues like Zeno Behavior.

4.3 Admissibility of Control Law

Zeno behavior refers to a situation in which the duration between two consecutive triggering instants approaches zero and the system undergoes infinite triggering instances within a finite duration. This situation results in excessive triggering of the switching mechanism, computational and communication overhead thereby degrading the system performance and reducing the efficiency of the system. To preclude Zeno behavior, it is required to ensure the existence of a finite inter execution time (Tabuada, 2007).

Defining an error term of the form

$$E = \left\| \hat{x}(t) - \hat{x}(t_k) \right\|; t \in [t_k, t_{k+1}) \tag{36}$$

Differentiating (36) and substituting (11) and (26) results in

$$\begin{aligned} \frac{dE}{dt} &= \frac{d\left\| \hat{x}(t) - \hat{x}(t_k) \right\|}{dt} \le \left\| \frac{d\hat{x}(t)}{dt} - \frac{d\hat{x}(t_k)}{dt} \right\| \le \left\| \frac{d\hat{x}(t)}{dt} \right\| \\ &\le \left\| A\hat{x}(t) + B\left(f\left(\hat{x}(t)\right) + u(t) \right) + m\hat{e}_p(t) \right\| \\ &\le \left\| A\hat{x}(t) + B\left(f\left(\hat{x}(t)\right) - Ke_d(t_k) - f(\hat{x}(t_k)) + \overset{n}{y}_d(t_k) \right) + m\hat{e}_p(t) \right\| \end{aligned} \tag{37}$$

$$\leq \left\| A\hat{x}(t) + B\left(Ke_d(t_k) + \overset{n}{y}_d(t_k) \right) + m\hat{e}_p(t) \right\| + \left\| \left(f\left(\hat{x}(t)\right) - f\left(\hat{x}(t_k)\right) \right) \right\|$$

$$\leq \left\| A\hat{x}(t) + B\left(Ke_d(t_k) + \overset{n}{y}_d(t_k) \right) + m\hat{e}_p(t) \right\| + L\left\| \hat{x}(t) - \hat{x}(t_k) \right\|$$

As all the closed loop signals are bounded, it is justified to assume that there exists a positive constant Φ such that

$$\left\| A\hat{x}(t) + B\left(Ke_d(t_k) + \overset{n}{y}_d(t_k) \right) + m\hat{e}_p(t) \right\| \leq \Phi \tag{38}$$

With (38), (37) it can be expressed as

$$\frac{dE}{dt} \leq \Phi + LE \tag{39}$$

The solution of (39) can be expressed as

$$\begin{aligned} E(t) &\leq \frac{\Phi}{L} e^{L(t-t_k)} - \frac{\Phi}{L} \\ (t - t_k) &\leq \frac{1}{L} \log_e \left\{ 1 + \frac{L}{\Phi} E(t) \right\} \end{aligned} \tag{40}$$

Substituting the boundary condition $t \to t_{k+1} : E(t) \to m_1^{1/2}$ in (40)

$$(t_{k+1} - t_k) \leq \frac{1}{L} \log_e \left\{ 1 + \frac{L}{\Phi} m_1^{1/2} \right\} \tag{41}$$

Now as $\frac{1}{L} \log_e \left\{ 1 + \frac{L}{\Phi} m_1^{1/2} \right\} < 0$

it means $(t_{k+1} - t_k) < 0$ (42)

The equation (41) indicates the existence of a finite lower bound on interexecution time.

Thus, Zeno behavior is successfully avoided.

The next section illustrates the simulation study carried out to validate the effectiveness of the control scheme.

5. Simulation

The following system dynamics is considered to conduct the simulation:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= f\left(x(t)\right) + u\left(t - \tau\right) : \forall t \leq 0 \\ y\left(t\right) &= x_1\left(t\right) \end{aligned} \tag{43}$$

where $f\left(x(t)\right) = 0.7\sin(x_1(t)x_2(t)) + 0.25x_1^2(t)x_2^2(t)$ and τ represents the channel induced delay. The system (43) belongs to the class of strict feedback systems (1) and satisfies all the stated assumptions. The following system conditions are taken for the simulation:

Initial condition: $x(0) = [0.3, 0]^T$

Channel induced delay: $\tau = 3\sec$

System (43) is further expressed as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(f+u) \\ y(t) &= Cx \end{aligned} \tag{44}$$

where $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

To compensate for the network induced delay, a predictor (9) of the following form is formulated as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\Big(f\left(\hat{x}(t)\right) + u(t)\Big) + me_p(t) \tag{45}$$

The predictor gain matrix is selected as: $m = \begin{bmatrix} 10.3 & 0 \\ 0 & 2.61 \end{bmatrix}^T$

The following event triggered state feedback law is formulated by using the predicted states

$$u(t) = u(t_k); u(t_k) = -Ke_d(t_k) - f(\hat{x}(t_k)) + \overset{n}{y}_d(t_k) \tag{46}$$

The controller term is formulated with the following parameter settings:

Gain settings: $K = [10.67, 1]^T$;desired tracking trajectory: $y_d(t) = \sin(t)$; event triggering threshold: $m_1 = 0.75$.

The results of the simulations conducted are shown in Figures 1,2, and 3. As revealed from the figures, the system states are bounded and closely track the desired trajectories; this reflects the effectiveness of the predictor model and predictor-based control term to alleviate the effects of the delay. The predictor states are in agreement with the system states and the prediction error converges to a residual set including the origin. The event-control term formulated is free from Zeno behavior with the minimum and maximum value of the inter-execution time equal to 0.2 sec and 2.3 sec respectively. On average there are 10 triggering instants over a span of 5 sec. These attributes seem feasible from a pragmatic point of view and reflect the effectiveness of the proposed scheme in terms of control on the network.

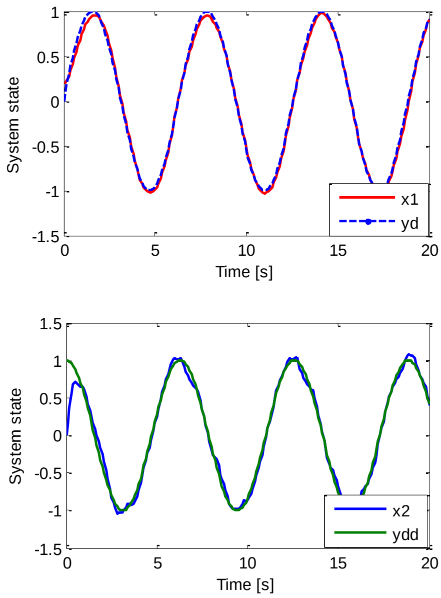

Figure 1. System states and corresponding desired trajectories: a. $x_1(t)$ and $y_d(t)$, b. $x_2(t)$ and $y_{dd}(t)$ (Source: Author's own research)

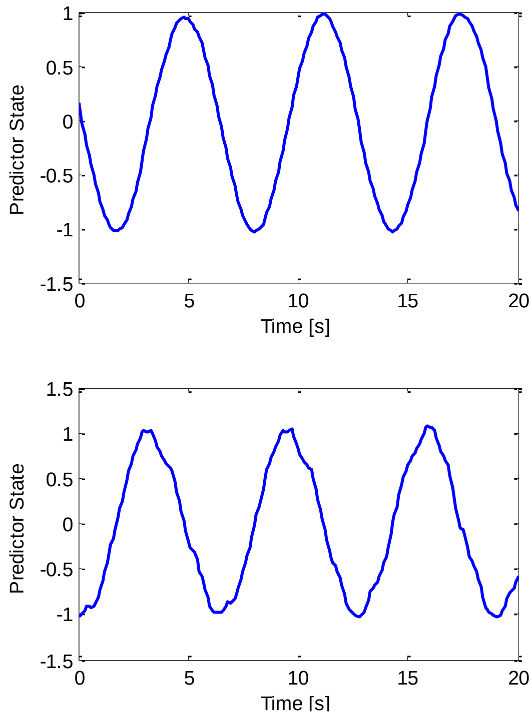

Figure 2. Predictor states: a. $\hat{x}_1(t)$, b. $\hat{x}_2(t)$ (Source: Author's own research)

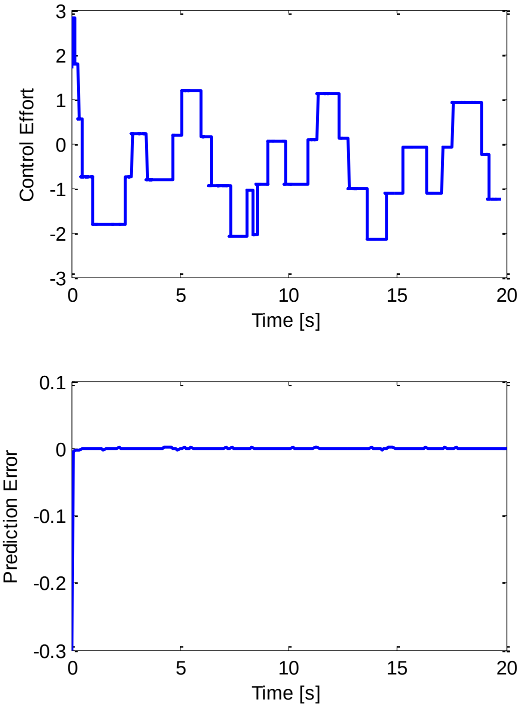

Figure 3. a. Control effort $u(t)$, b. Prediction error $e_p(t)$ (Source: Author's own research)

5. Conclusion

This paper addresses the issue of designing a predictor-based event-triggered control policy for cyber physical systems operating under the conditions of network delay and limited network resources. A predictor-based control policy is formulated, and its effectiveness is analyzed under the conditions of arbitrary time delay. Event triggering allows the effective sharing of the network resources and also reduces the computational efforts of the controller. Lyapunov-based validations have been developed to testify the boundedness of closed loop signals and the Zeno free behavior of the control term. The results of the simulation illustrate the promising performance of a control scheme in the presence of the channel delay and limited resources.

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