Graph-based analysis of the Bucharest transport network

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Abstract: Thanks to the European recovery instrument "NGEU", there are several opportunities to finance new infrastructures in the EU member states. For this reason, based on the graph theory, a preliminary analysis of the transport network in Bucharest is proposed, analysis which may be of interest in a future study regarding some funding proposals. This network includes the entire transport system of Romania's capital. The main three contributions of this article are the following: the analysis of the structural vulnerabilities of the network, the study of the global geometric properties of this network, and the identification of the main transport stations, by using several centrality indices, the existence of some correlations between these indices being highlighted.

Keywords: Network analysis, Network geometry, Centrality indices, Graph theory.

Analiza bazată pe grafuri a rețelei transporturilor în Bucureşti

Rezumat: Datorită instrumentului european de redresare „NGEU'', există diferite oportunități de finanțare a unor infrastructuri noi în statele membre UE. Din acest motiv, pornind de la teoria grafurilor, se propune o analiză preliminară a rețelei transporturilor din Bucureşti, analiză ce ar putea fi de interes într-un viitor studiu al unor propuneri de finanțare. Această rețea include întregul sistem de transport al capitalei României. Principalele trei contribuții ale acestui articol sunt următoarele: analiza vulnerabilităților structurale ale rețelei, studiul proprietăților geometrice globale ale acestei rețele şi identificarea stațiilor principale de transport, prin folosirea mai multor indici de centralitate, fiind evidențiată existența unor corelații între aceşti indici.

Cuvinte cheie: Analiza rețelelor, Geometria rețelelor, Indicii de centralitate, Teoria grafurilor.

1. Introduction

Bucharest has the fourth largest transportation system in Europe. Its modernization is often cited as a top objective by the Mayorship of Bucharest. Recently, there has been increased funding for the renovation of such critical infrastructures, including the program ,,Anghel Saligny" in Romania, and the European Union Recovery Instrument ,,NGEU". In this paper, existing relations between the public transport stations in Bucharest are examined, using tools from Graph Theory (Barabási, 2016). The scope of this study is to identify inherent vulnerabilities and some important features of the Bucharest lines of transport, to be taken into account in future funding decisions. Other crucial aspects, such as geographical data, population density and the degradation of current infrastructures and fleets are not considered in the present work.

There is a broad literature on transport networks in Romania and around the world (Bordea & Anghel, 2005; Mhalla & Dutilleul, 2023). In particular, the road network of Bucharest has been extensively studied in the context of natural hazards, such as earthquakes, and their impact on transportation (Ruscă et al., 2014; Toma-Danila, 2018; Toma-Danila et al., 2020). In the road network of Bucharest, vertices and edges represent road intersections and individual road segments, respectively (there are 50,412 edges). By contrast, the network generated by public transport lines: with vertices for stations, and edges for direct connections, has received less attention. Transport sub-networks in Bucharest, including its subway network (Dragu et al., 2011; Ştefănică et al., 2013) and geographic information on its tram network (Andrei & Luca, 2021), have been studied in separate prior works. A SWOT analysis for the whole public transport system in Bucharest was proposed in (Bugheanu, 2015). However, in these previous studies, graph-based metrics for the

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Bucharest transport network and its sub-networks have not been analyzed, or only the study of some connectivity properties was considered.

Contributions. In this paper, a graph representation of the Bucharest transport network is analyzed. There are two different transport operators in Romania's capital, namely: Metrorex for its subway system, and the Bucharest Transport Company – STB for buses, trolleybuses, and trams. Information on both sub-networks is publicly available on the respective websites of their operators, but it can also be found on applications such as Moovit (2024), which is more suitable for data scraping techniques. For each operator, a graph is created: where nodes represent the stations, and there are links representing the direct connections between the stations. This way, each line is mapped to either a path or a cycle in the graph representation. There are 5 and 107 lines in both sub-networks, respectively. Stations in the same sub-network are identified by their names. As such, a station can be mapped to one or more physical locations. In some rare cases (8 cases encountered for the STB sub-network), a line can have two consecutive stops with the same name. This results in a loop in the graph representation. Similarly, there are multiple edges, because of different lines sharing a common road. Multiple edges are removed from the final representations. Finally, both sub-networks are combined in one graph, as follows: stations are identified by their names, and there is an additional set of links that represent possible transfers - by foot - between subway stations and ground stations. The links for transfers were deduced from a manual inspection of the geographic neighbourhood of all subway stations. Overall, the resulting graph (see Figure 1) has 887 vertices and 1,482 edges.

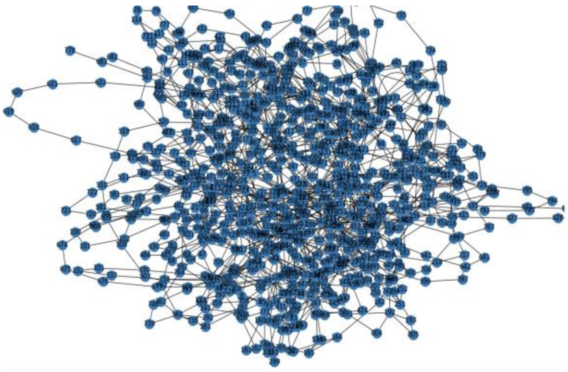

Figure 1. Illustration of Bucharest transport network (own research)

The edges of the graph are left unoriented. Indeed, there are not so many cases in which the line between two terminal stations differs in its both directions. Furthermore, the variable length of direct connections between the stations was not accounted for (i.e. the graph is unweighted). From a user's perspective, the *hop distance* (number of transport connections) is considered, by opposition to the total travel time. Edge directions and weights could be included in a future work.

The remainder of this paper is as follows. Section 2 presents an analysis of the structural vulnerabilities of the network. Section 3 describes the global geometric properties of the network. In Section 4, the rankings of the nodes obtained using different centrality indices are compared. Finally, Section 5 outlines the conclusion of this paper.

2. Structure and vulnerabilities

Two classic distributions for networks (for degrees and for hop distances, respectively) are reported and briefly discussed in sub-sections 2.1. and 2.2. In sub-section 2.3., potential structural vulnerabilities are evaluated using connectivity properties.

2.1. Degree distribution

The degree distribution is included in Table 1. See Figure 2 for an illustration of the number of vertices of a given degree.

Table 1. Degree distribution for the Bucharest transport network

degree	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**15**	**18**	**20**	**24**
number of nodes	8	440	167	103	59	40	27	17	9	5	4	1	3	1	1	1	1

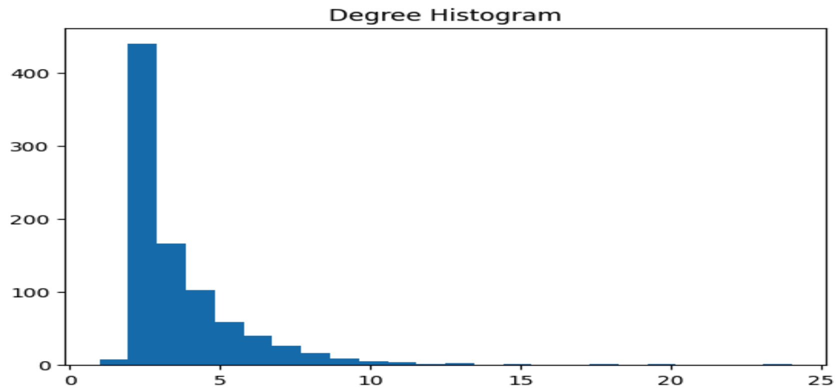

Figure 2. Number of vertices of a given degree (own research)

It can be observed that half of all stations belong to a unique public transport line (their degree is two at most), with eight of these stations being terminal points (their degree is one). The average degree is 3.34. The latter suggests that an arbitrary station should appear on at most two different lines. However, there are also some hubs in Bucharest city center, as it can be checked from the *h-index*: the maximum value h such that there are h nodes of degree at least h. There are 12 nodes whose degree is at least eleven, and so the h-index of the network equals 11. At present, it is unclear which distribution is followed by the degree. Experiments, using standard regression techniques, suggest that it is not a power-law distribution.

The *clustering coefficient* of a node measures the density of edges in its neighbourhood. In the Bucharest transport network, the maximum clique size is four. Furthermore, there are only two cliques of size four, and 145 triangles. Hence, the clustering coefficient of most nodes is null (for 72% of the nodes), or very close to zero, which is as expected in a public transport network. In particular, the maximum value 1 is reached for only 27 nodes. Similarly, the *k-core* of a graph is its largest subgraph, excluding loops, whose minimum degree is at least k. The *core number* of a node is equal to the largest k such that it belongs to the k-core. For every station in the Bucharest transport network, the core number ranges between 1 and 3. The only vertices of core number 1 are the 8 terminal points (degree-one nodes). There is 19% of all nodes with core number 3. The nonexistence of a k-core, for k > 3, is evidence against a core/periphery structure (Seidman, 1983).

2.2. Hop distance distribution

The hop distance distribution is reported in Table 2. See Figure 3 for an illustration of the number of pairs of vertices at a given distance.

Table 2. Distance distribution

hop distance	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
number of pairs	1,474	4,601	11,986	26,922	50,482	74,290	82,312	68,541
hop distance	**9**	**10**	**11**	**12**	**13**	**14**	**15**	
number of pairs	42,596	20,053	7,122	2,012	461	79	10	

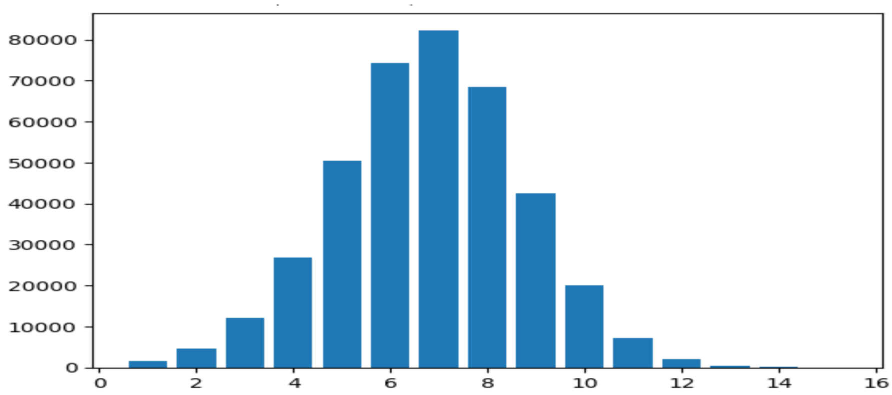

Figure 3. Number of pairs of vertices at a given distance (own research)

It can be observed that the number of pairs of vertices at unit distance is lower than the number of edges, which is because there are loops in the network. The maximum hop distance and the average hop distance are 15 and 6.8, respectively. Therefore, any station can be reached after fifteen stops at most, and the average number of stops on a shortest trip is around seven.

The *eccentricity* of a node is its largest hop distance to the other nodes. The eccentricity distribution is reported in Table 3.

Table 3. Eccentricity distribution

eccentricity	**9**	**10**	**11**	**12**	**13**	**14**	**15**
number of nodes	22	134	252	258	168	45	8

The *radius* (minimum eccentricity) is R = 9. As it was mentioned previously, the *diameter* (maximum hop distance) is D = 15. In particular, the tree-like relation D = 2R – 3 holds. This relation is coherent with a situation in which for most nodes v in the network, their shortest trip to a furthest station should go through a center node (i.e. a node of minimum eccentricity), similarly to what happens in tree networks. Since there are few center nodes (24), a high level of congestion at some of these nodes may be anticipated. Further evidence for the inherent vulnerability of the Bucharest public transport system to congestion is given in Section 3, where the *hyperbolicity* of a graph is introduced.

2.3. Separators and cuts

It has been reported in the literature that the tram sub-network of Bucharest is disconnected (Andrei & Luca, 2021). By contrast, in the Bucharest transport network, all types of public transports are taken into account, and the resulting graph is connected. A *cut vertex* is a vertex whose removal disconnects the graph. Since there are eight terminal points (degree-one nodes), there are as many cut vertices. However, these are the only cut vertices in the network, thus showing that the network is essentially biconnected (up to the removal of terminal points).

The *treewidth* is a well-studied connectivity parameter in Graph Theory. By and large, the treewidth of a network is the least k such that it can be recursively disconnected using separators of size at most k, in a tree-like fashion (Robertson & Seymour, 1984). Calculating the treewidth is computationally expensive, but there exist efficient heuristics such as the Minimum Degree Algorithm (Tinney & Walker, 1967). Applied to the Bucharest transport network, the output of this heuristic is a treewidth upper bound of 43 (an improved upper bound of 40 could be achieved using the treewidth_min_fill_in function of the Python NetworkX library). The latter result suggests a good resilience of the network, in the sense that a relatively large number of nodes need to be removed in order to disconnect the network in two large halves (some small sections of the network are much easier to disconnect, though).

3. Underlying geometry

Roughly, some important observed properties of real networks, such as scale-freeness, can be linked to the special features of an abstract manifold, with which the topological architecture of a network is identified. The latter is the premise of *Network geometry*: an emerging sub-field of Network science, for which a good survey can be found in (Boguňá et al., 2021). Overall, these geometric aspects are relevant in the study of traffic congestion (Chepoi et al., 2017), and geometric embeddings of networks that are used for greedy routing schemes (Kleinberg, 2007) - with applications to traffic and transportation planning - and for Machine Learning (Gu et al., 2021). There are also algorithmic applications for such geometric features to the design of efficient algorithms for facility location problems on networks (Ducoffe et al., 2022). In what follows, three global parameters of graph metrics are discussed and reported.

A graph is called *planar* if it can be drawn in the Euclidean plane in such a way that its edges only intersect at their common endpoints. The properties discussed further on are not related to planarity. In fact, the experiments carried out show that the Bucharest transport network is far from planar. Indeed, it should be noted that a graph is planar if and only if it does not contain a so-called Kuratowski subgraph (a subdivision of the complete graph K_5 or of the complete bipartite graph $K_{3,3}$). Furthermore, the left-right planarity test can be used in order to output a Kuratowski subgraph in a nonplanar graph (De Fraysseix & Rosenstiel, 1982). By iteratively removing such output until the graph becomes empty or planar, a collection of node-disjoint Kuratowski subgraphs can be greedily computed. For the Bucharest transport network, a collection of 21 disjoint Kuratowski subgraphs was computed, the union of which contains 46% of all the vertices. This could be expected since both the subway transport system and the ground transport system are considered, leading to several edge intersections in the natural embedding of the network in the plane according to the geographic coordinates of the stations.

3.1. Network curvature

The notion of global curvature can be defined as follows for any network (in fact, for any metric space): a graph is called *δ-hyperbolic* if for every 4-tuple u,v,x,y of vertices, the two largest distance-sums amongst d(u,v)+d(x,y), d(u,x)+d(v,y), and d(u,y)+d(v,x) differ by at most 2δ. The hyperbolicity of a graph is the lowest value δ such that it is δ-hyperbolic (Gromov, 1987). In particular, trees are 0-hyperbolic, and the hyperbolicity is a measure of how similar an arbitrary graph metric is to tree metrics. The hyperbolicity of a network can be computed by a direct inspection of all its 4-tuples of vertices. However, this naive algorithm did not terminate on the Bucharest transport network after one day of computation. A faster practical algorithm was proposed by Cohen et al. (2015), thanks to which it could be verified that its hyperbolicity was equal to 4. By comparison, the maximum theoretical value for the hyperbolicity in a network of diameter D = 15 is 7. A network is called *strongly hyperbolic* if its hyperbolicity is upper bounded by the base-two logarithm of its diameter (Alrasheed & Dragan, 2017). The Bucharest transport network is strongly hyperbolic, which is evidence for inherent core congestion in the system (Chepoi et al., 2017).

3.2. Vapnik-Chervonenkis dimension

The Vapnik-Chervonenkis dimension (for short, VC-dimension) is a well-studied parameter for measuring the complexity of set systems, including that of concept classes in Machine Learning (Blumer et al., 1989) and, more relevant to this work, of graphs. For every vertex v, its *closed neighbourhood* N[v] contains v and all its neighbours. A vertex subset X is called *shattered* if and only if for every of its subsets Y, there exists a vertex v such that X ∩ N[v] = Y. The VC-dimension of a network is the maximum cardinality of its shattered subsets. Efficient algorithms are known for some facility location problems on networks of bounded VC-dimension (Ducoffe et al., 2022), with applications to transport networks. It follows from a simple counting argument that in any shattered subset of d vertices, the degree of each vertex must be at least $2^{d-1} – 1$. In the Bucharest transport network, the largest d such that there exist d vertices of degree at least $2^{d-1} – 1$

is d = 4, which is an upper bound to its VC-dimension. By comparison, the maximum theoretical value d_{max} for the VC-dimension of an n-node network is at most log(n), which for n = 887 leads to d_{max} = 9.

3.3. Helly number

Another well-studied invariant for measuring the complexity of set systems is the so-called Helly number. For networks, the Helly number is the least integer k such that, in every family F of k-wise intersecting disks (of arbitrary centers and radii), there is a vertex contained in every disk of F. Since the Euclidean space R^d has Helly number d+1, the Helly number of a network may be a useful indicator for its embedding in an Euclidean space of smallest dimension, with applications to the design of routing protocols, and so, in traffic and transportation planning. Furthermore, there exist efficient algorithms for some facility location problems on networks of bounded Helly number (Ducoffe, 2023). For computing the Helly number of a network, Gilmore's condition can be used: more specifically, the Helly number is at most k if and only if for every (k+1)-set S of vertices, the family of all disks with at least k vertices in S has a nonempty common intersection (Berge, 1973). The runtime of Gilmore's algorithm grows exponentially with k. It could not be run on the Bucharest transport network, already for k < 5. Therefore, the following strategy was used in order to quickly exclude small values of k: a random sample S of k+1 vertices is drawn, a random sample U of at most $\sqrt{n}$ vertices is drawn, then it is checked whether all disks with center in U and at least k vertices in S have a nonempty intersection. Doing so, it was possible to assert that the Helly number of the Bucharest transport network must be at least 5. Gilmore's condition was tested in full for k=5 in order to confirm that the Helly number of this network is indeed 5.

4. Centrality indices

Centrality indices are partial rankings of the nodes, in order to estimate their relative importance in a network according to various criteria (Koschützki et al., 2005). In this section, some classic centrality indices are introduced. These centrality indices were computed for each node in the Bucharest transport network, in order to identify the main stations. Statistical comparisons between the results obtained, for different centrality indices, are reported.

4.1. Definitions

The following centrality indices have been considered in this study:

- Degree centrality: nodes are ranked according to their degrees. In particular, the most important nodes are those of maximum degree;
- Harary centrality: nodes are ranked according to the inverse of their eccentricities. In particular, the most important nodes are the centers, whose eccentricity equals the radius;
- Closeness centrality: nodes are ranked according to the inverse of their average distance to the other nodes. The most important nodes with respect to closeness centrality are sometimes called medians;
- Betweenness centrality: The betweenness of a node v is the sum, over every pair of vertices (x,y) such that $x \neq v \neq y$, of the fraction of shortest (x,y)-paths going through v. Nodes are ranked according to their betweenness values;
- Katz centrality: Every walk of length k has weight α^k, for some $0 < \alpha < 1$, and the centrality score of a node v is defined as the sum, over all k, of the weights of all its walks of length k to the other nodes. In particular, all walks are taken into account, not just the shortest paths;
- Eigenvector centrality: nodes are ranked according to the coordinates of the unique nonnegative eigenvector of unit 1-norm that is associated to the largest eigenvalue of

the adjacency matrix. Very roughly, the latter is a refinement of degree centrality, in which the higher the degree of a neighbour, the more it contributes to the centrality score.

Since the importance of a node is measured differently for these centrality indices, it is not clear a priori whether the rankings obtained for the nodes should be similar.

4.2. Results

The centrality indices of all nodes in the Bucharest transport network were computed. The rankings obtained for the nodes were compared using Pearson correlation coefficient (Pearson, 1895). These results are reported in Table 4.

Table 4. Statistical comparison between centrality indices, using Pearson correlation coefficients

	Degree	**Harary**	**Closeness**	**Betweenness**	**Katz**	**Eigenvector**
Degree		p = .41	p = .53	p = .78	p = .94	p = .54
Harary	p = .41		p = .86	p = .42	p = .52	p = .36
Closeness	p = .53	p = .86		p = .58	p = .68	p = .55
Betweenness	p = .78	p = .42	p = .58		p = .8	p = .55
Katz	p = .94	p = .52	p = .68	p = .8		p = .72
Eigenvector	p = .54	p = .36	p = .55	p = .55	p = .72	

All p-values (not reported in the table) were below 2.4E-28. Overall, all the centrality indices considered are positively correlated ($p > 0$). There is a moderate degree of correlation in all cases ($p \geq .3$) and a strong correlation in most cases ($p \geq .5$). In particular, there is almost a perfect correlation between degree centrality and Katz centrality. Roughly, it implies that all rankings considered for the nodes can be approximated by a simple inspection of the degree sequence.

5. Conclusion

The conducted experiments suggest that the Bucharest transport network is quite resilient (i.e. the treewidth is large), but that it is inherently vulnerable to traffic congestion (i.e. it has bounded hyperbolicity). A few high-degree hubs are identified, with evidence that most of the traffic is inward-oriented, as supported by the positive correlations between various centrality indices. Finally, although this network is far from planar, as it could be expected, it has a nontrivial geometry, as supported by the boundedness of some abstract geometric parameters (VC-dimension, Helly number). The latter could be exploited for traffic and transportation planning, and algorithmically in order to solve some facility location problems on the network more efficiently. Recently, the security aspects of some transport systems have been considered (Predescu et al., 2023). These aspects could be analyzed for the Bucharest transport network in a future work.

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REFERENCES

Alrasheed, H. & Dragan, F. F. (2017) Core–periphery models for graphs based on their δ-hyperbolicity: An example using biological networks. *Journal of Algorithms & Computational Technology*. 11(1), 40-57. doi: 10.1177/1748301816665519.

Andrei, L. & Luca, O. (2021) Open Tools for Analysis of Elements Related to Public Transport Performance. Case Study: Tram Network in Bucharest. *Applied Sciences*. 11(21), 10346. doi: 10.3390/app112110346.

Barabási, A. L. (2016) *Network Science*. Cambridge, Cambridge University Press.

Berge, C. (1973) *Graphs and Hypergraphs*. Amsterdam, North-Holland Publishing Company.

Blumer, A., Ehrenfeucht, A., Haussler, D. & Warmuth, M. K. (1989) Learnability and the Vapnik-Chervonenkis dimension. *Journal of the ACM (JACM)*. 36(4), 929-965. doi: 10.1145/76359.76371.

Boguna, M., Bonamassa, I., De Domenico, M., Havlin, S., Krioukov, D. & Serrano, M. Á. (2021) Network geometry. *Nature Reviews Physics*. 3(2), 114-135. doi: 10.1038/s42254-020-00264-4.

Bordea, V. & Anghel, R. (2005) Maximization for Maritime Transport Networks. *Romanian Journal of Information Technology and Automatic Control*. 15(3), 5-14.

Bugheanu, A. M. (2015) SWOT analysis of public transport system in Bucharest. *Management Research and Practice*. 7(1), 14-31.

Chepoi, V., Dragan, F. F. & Vaxes, Y. (2017) Core congestion is inherent in hyperbolic networks. In: Klein, P. N. (ed.) *Proceedings of the 28th Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2017, 16-19 January, 2017, Barcelona, Spain*. Philadelphia, Pennsylvania, Society for Industrial and Applied Mathematics. pp. 2264-2279.

Cohen, N., Coudert, D. & Lancin, A. (2015) On computing the Gromov hyperbolicity. *Journal of Experimental Algorithmics (JEA).* 20(1), 1-18. doi: 10.1145/2780652.

De Fraysseix, H. & Rosenstiehl, P. (1982). A depth-first-search characterization of planarity. *North-Holland Mathematics Studies.* 62, 75-80. doi: 10.1016/S0304-0208(08)73550-3.

Dragu, V., Ştefănică, C. & Burciu, Ș. (2011) Accessibility study in regard to Bucharest underground network. *University Politechnica of Bucharest Scientific Bulletin, Series D*. 73(1), 221-236.

Ducoffe, G. (2023) Distance problems within Helly graphs and k-Helly graphs. *Theoretical Computer Science*. 946, 113690. doi: 10.1016/j.tcs.2023.113690.

Ducoffe, G., Habib, M. & Viennot, L. (2022) Diameter, Eccentricities and Distance Oracle Computations on H-Minor Free Graphs and Graphs of Bounded (Distance) Vapnik–Chervonenkis Dimension. *SIAM Journal on Computing*. 51(5), 1506-1534. doi: 10.1137/20M136551X.

Gromov, M. (1987) Hyperbolic groups. In: Gersten, S. M. (ed.) *Essays in group theory.* New York, NY, Springer New York, pp. 75-263.

Gu, W., Tandon, A., Ahn, Y. Y. & Radicchi, F. (2021) Principled approach to the selection of the embedding dimension of networks. *Nature Communications*. 12(1), 3772. doi: 10.1038/s41467-021-23795-5.

Kleinberg, R. (2007) Geographic routing using hyperbolic space. In: Baldwin, R. (ed.) *Proceedings of the 26th IEEE International Conference on Computer Communications, INFOCOM 2007, 6-12 May 2007, Anchorage, Alaska*. Piscataway, New Jersey. IEEE. pp. 1902-1909.

Koschützki, D., Lehmann, K. A., Peeters, L., Richter, S., Tenfelde-Podehl, D. & Zlotowski, O. (2005) Centrality indices. In: Brandes, U. & Erlebach, T. (eds.) *Network analysis: methodological foundations (Lecture Notes in Computer Science, vol. 3418)*. Berlin, Springer, pp. 16-61. doi: 10.1007/978-3-540-31955-9_3.

Mhalla, A. & Dutilleul, S. C. (2023) Algorithms for the Computation of Passive Robustness Margins in Railway Transport Systems**.** *Studies in Informatics and Control*. 32(2), 85-92. doi: 10.24846/v32i2y202308.

Moovit (2024) *Bucuresti Public Transit Trip Planner*. https://moovitapp.com/index/en/public_transit-Bucure%C8%99ti-2960 [Accessed: 19th March 2024.

Pearson, K. (1895) VII. Note on regression and inheritance in the case of two parents. *Proceedings of the Royal Society of London*. 58 (347-352), 240-242. doi: 10.1098/rspl.1895.0041.

Predescu, S. G., Savu, D. & Badea, V. E. (2023) Cybersecurity in the Railway Sector**.** *Romanian Cyber Security Journal*. 5(1), 95-104. doi: 10.54851/v4i2y2022010.

Robertson, N. & Seymour, P. D. (1984) Graph minors. III. Planar tree-width. *Journal of Combinatorial Theory, Series B*. 36(1), 49-64. doi: 10.1016/0095-8956(84)90013-3.

Ruscă, F. V., Roşca, E., Ruscă, A., Dragu, V. & Burciu, Ș. (2014) Social inequity induced by Bucharest road network vulnerability. In: Voicu, M. (ed.) *Advances in Automatic Control (The Springer International Series in Engineering and Computer Science, SECS, vol. 754)*. New York, Springer. pp. 221-225.

Seidman, S. B. (1983) Network structure and minimum degree. *Social networks*. 5(3), 269-287. doi: 10.1016/0378-8733(83)90028-X.

Ştefãnică, C., Dragu, V., Burciu, S. & Ilie, A. (2013) Connections between Bucharest underground and rail networks. In: Marascu-Klein, V. & Ciontu, M. (eds.) *Proceedings of the 15th International Conference on Automatic Control, Modelling, and Simulation, ACMOS 2013, 1-3 June 2013, Braşov, Romania*. WSEAS. pp. 92-97.

Tinney, W. F. & Walker, J. W. (1967) Direct solutions of sparse network equations by optimally ordered triangular factorization. *Proceedings of the IEEE*. 55(11), 1801-1809. doi: 10.1109/PROC.1967.6011.

Toma-Danila, D. (2018) A GIS framework for evaluating the implications of urban road network failure due to earthquakes: Bucharest (Romania) case study. *Natural Hazards*. 93 (Suppl 1), 97-111. doi: 10.1007/s11069-017-3069-y.

Toma-Danila, D., Armas, I. & Tiganescu, A. (2020) Network-risk: an open GIS toolbox for estimating the implications of transportation network damage due to natural hazards, tested for Bucharest, Romania. *Natural Hazards and Earth System Sciences*. 20(5), 1421-1439. doi: 10.5194/nhess-20-1421-2020.

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